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SPECTROGRAPHIC OBSERVATIONS OF THE ROTATION OF THE SUN[1]

By WALTER S. ADAMS

The spectroscopic study of the rotation period of the sun's reversing layer has hitherto been confined to visual measures of lines in the less refrangible part of the spectrum. In 1890 Dunér published his classical research upon the subject, including in his discussion results covering the period of three years from 1887 to 1889. Later he supplemented these values with observations made during the years 1899, 1900, and 1901.[2] During the last of these years Halm began the same investigation at Edinburgh, using a fixed horizontal spectroscope and a heliometer to bring the images of the opposite limbs of the sun upon the slit of his instrument. He has since published determinations extending up to 1906[3] and his measures indicate results of the highest accuracy, the probable error for a single observation falling considerably below that of Dunér. In both of these investigations the lines employed were the same, and consisted of a pair of iron lines in the red region of the spectrum having the wavelengths 6301.72 and 6302.71 on Rowland's scale. The desirability of extending the research to other elements and of employing the obvious advantages of the photographic method as soon as suitable apparatus was available was early realized by Professor Hale at this

[1] *Contributions from the Solar Observatory*, No. 20.

[2] *Astronomische Nachrichten*, **167**, 167, 1905.

[3] *Idib.*, **173**, 273–295, 1907.

observatory, and it was at his suggestion that this investigation was commenced.

It is clear that a satisfactory photographic study of the displacements at the sun's limb requires a solar image of considerable size combined with a spectrograph of high dispersion, and sufficient focal length to give full photographic resolution. Both of these are available with the Snow telescope. The concave mirror of this instrument forms an image upon the slit of the spectrograph about 6.7 inches (17.0 cm) in diameter. The spectrograph is of the Littrow, or auto-collimation, type, with a lens 4 inches (10.2 cm) in diameter and 18 feet (5.5 m) focal length employed in conjunction with a plane grating of the same aperture. This grating, which by the kindness of Professor Frost of the Yerkes Observatory has been loaned to the Solar Observatory, is one of the early Rowland gratings, and has 14,438 lines to the inch (570 lines to the mm). It is exceptionally bright in both the third and fourth orders, and gives excellent definition in spite of the great focal length of the spectrograph. The spectra used in this investigation have all been taken in the fourth order, the advantages of the larger scale more than offsetting the greater width of the spectrum lines. The linear scale of the instrument in this order at λ 4200 is about 1 mm = 0.71 Ångström unit.

The apparatus used to bring the opposite limbs of the sun together on the slit is an adaptation of that first employed by Langley and afterward used by Dunér in his well-known investigations. A pair of small diagonal prisms is mounted on each of two rotating brass arms. The first of these prisms is placed at the outer end of the arm, its mean distance from the center of rotation corresponding to the mean radius of the sun's image, and is capable of adjustment along the arm to correspond with the varying size of the image. A second prism, which receives the beam from the first and reflects it upon the slit, tapers at the end to a width of about 0.5 mm, and is mounted with a point slightly inside the center of this end immediately above the center of rotation of the brass arm. The distance between the edges of the prisms on the two arms is about 0.25 mm, which accordingly represents the distance on the photographic plate between the spectra of the two opposite limbs. The two arms are rotated on a brass frame, and are provided at the ends with pointers by means of

which readings are made on a silver position-circle, graduated to half-degrees, and capable of being estimated to tenths. The whole apparatus is mounted upon a brass casting which rests upon a large bracket below the slit of the spectrograph, its position being accurately defined by two taper pins which enter this bracket.

In adjusting this instrument previous to beginning upon the series of observations, great care was taken to secure equal and uniform illumination of the grating surface from the two sets of prisms at all position angles. The simplest method of doing this was to occult the images of the two limbs in succession, and to examine visually from the position of the photographic plate the character of the illumination of the grating. With a narrow slit and comparatively weak illumination this method gives good results. It has, however, been supplemented with photographic tests, the illumination of the collimating lens being photographed on sections of sensitized paper pressed against the rear surface of its cell. The adjustment once made, it has been found necessary to change it on only one occasion, when, owing to a fracture, one of the small diagonal prisms had to be replaced. Since the ratio of aperture to focal length is 1 to 54 in the case of the collimating lens, and 1 to 30 for the image-forming telescope, it is clear that the margin of safety is considerable. It has, however, been the practice to examine visually the character of the illumination previous to each exposure; and it is needless to add that a further valuable check is furnished by the relative density of the pair of spectra upon the photographic plate.

The procedure followed in making the observations has been as follows. The rotation attachment is set in place upon the bracket beneath the slit of the spectrograph, and the image of the sun focused upon the position circle at its edge. The clock driving the coelostat is then stopped, and a spot or other well-defined point upon the sun's surface is allowed to transit across the circle, readings being made at both points of crossing. The mean of three or four such observations, which rarely show a range of more than $0^\circ.3$, is used as the line of reference for the determination of the heliographic positions. The pointers on the arms carrying the diagonal prisms are then set at the proper reading of the position circle, the character of the illumination of the grating surface is examined, and the exposure made. The

position angle is then changed and the process repeated. On the majority of the plates an exposure has been made for every 15° of latitude between 0° and 75°. This has been done in order to obtain results directly comparable with those of Dunér. A considerable number of intermediate points have been used, however, particularly in high latitudes. At the close of the set of exposures a second series of transits of the spot across the position circle is taken.

The selection of the region of the spectrum most suitable for the work has given considerable difficulty on account of the necessity of securing a sufficiently varied list of lines within a comparatively short extent of spectrum. The portion finally chosen is that extending from λ 4190 to λ 4300. This includes a part of the extremely rich G region, and has the additional advantage of containing the head of the violet carbon fluting, some lines of which it is most desirable to use in the investigation. Another determining feature was the fact that the maximum of sensitiveness of the Seed "process plate" lies not far from this point. This plate has always proved very satisfactory for spectrum work in the blue and violet regions, showing a fine grain and excellent contrast, while at the same time it is appreciably more rapid than the ordinary transparency or lantern-slide plates. The following list of lines was finally adopted:

λ	Element	Intensity	Remarks
4196.699	*La*	2	Much weakened at limb.
4197.257	*C*	2	Slightly weakened at limb
4203.730	*Cr*	2	Strengthened and widened at limb
4209.144	*Zr*	1	Weakened at limb
4216.136	*C*	1	Weakened at limb
4220.509	*Fe*	3	Slightly strengthened at limb. Chromospheric line
4232.887	*Fe*	2	Much strengthened at limb
4257.815	*Mn*	2	Probably weakened at limb
4258.477	*Fe*	2	Much strengthened at limb. Much strengthened in sun-spots
4265.418	*Fe*	2	Slightly weakened at limb
4266.081	*Mn*	2	Perhaps weakened at limb
4268.915	*Fe*	2	Slightly weakened at limb
4276.836	–*Zr*	2	Weakened at limb
4284.838	*Ni*	1	Slightly weakened at limb
4287.566	*Ti*	1	Slightly strengthened at limb. Strengthened in sun-spots
4288.310	*Ti, Fe*	1	Widened at limb
4290.377	*Ti*	2	Slightly weakened at limb. Enhanced line of *Ti*
4290.542	*Fe*	1	Probably weakened at limb
4291.630	*Fe*	2	Much strengthened at limb. Strengthened in sun-spots
4294.936	*Zr*	2	Probably weakened at limb

At the time at which this list of lines was selected the remarkable differences between the spectrum of the center and that of the limb of the sun were not known.[1] It was, however, noted that the lines upon the plates appeared in general rather diffuse and "matt," to use the German expression, and the exposure times were much longer than was to be expected from exposures made on the disk of the sun without auxiliary apparatus. A part of this effect was ascribed to the fact that the light was obliged to traverse some 3 inches (76 mm) of glass in passing through the diagonal prisms. The true cause, however, was not understood until the investigations of Professor Hale and myself showed the radical difference in character and intensity of the spectra of the two parts of the sun's image.

The necessity for selecting only the lines best adapted for measurement has, of course, excluded many interesting lines from the above list, but those given may be regarded as reasonably comprehensive as regards the elements represented, and their behavior at the limb and the center of the sun. The line due to lanthanum is included on account of the high atomic weight of this element, and a similar reason holds for the three lines of zirconium, though in less degree. Carbon is of great interest on account of its position in the chromosphere, and is represented by two lines. The remaining lines are divided among the more important solar elements, iron naturally occupying the most prominent position.

The series of plates amounting to 44 in number included in this discussion was begun in May 1906, and extends to June 1907, a period of nearly fourteen months. Though not distributed uniformly throughout this time they cover the period fairly well with the exception of the interval from July to October, 1906. During these months it was not possible to secure observations on account of the breaking of one of the small central diagonal prisms. In selecting the plates to be measured, only such were chosen as were taken on days when the sky was suitably transparent, and no daylight spectrum was superposed upon the spectra of the two limbs. This point was usually tested by direct visual observation, in the same way as was done by Halm in his series of visual measures in the less refrangible region of the spectrum.

[1] Hale and Adams, *Contributions from the Solar Observatory*, No. 17; *Astrophysical Journal*, 25, 215–225, 1907.

The computation of the heliographic latitudes of the observed points has been made for the most part with the use of De La Rue's reduction tables. These give with the sun's longitude as an argument the position angle of the sun's axis in reference to the north point, and the heliographic latitude of the earth. Since we know the position-circle reading of the point under observation as well as that of the east and west line, the position angle from the north point is known at once, and the computation of the latitude is made simply. For setting the position circle during the observations the table for the position angle of the sun's axis given in the *Companion to the Observatory* has been found very useful. The angle, by the secant of which it is necessary to multiply the observed velocities in order to correct for the departure of the sun's pole from its visible edge, has been taken from the table given by Dunér, except for high latitudes, in which case it has been computed directly. The further correction to be applied for the distance inside the sun's edge from which the light which passes through the slit of the spectrograph has been taken, is found as follows. With the almanac value of the sun's diameter and the scale-setting of the concave mirror of the telescope the value of the linear diameter of the sun's image is computed. The distance between the small windows through which the diagonal prisms receive the light being accurately known, the factor required is readily derived. In practice it has been found preferable to change this distance occasionally rather than to attempt to keep at a fixed distance from the sun's edge as the diameter varied.

The greater part of the plates have been measured by Miss Lasby of the Computing Division upon the 150 mm measuring machine built by Toepfer. The screw of this instrument has a pitch of 0.5 mm, and the divided head may be read to 0.5 μ. A series of measures upon a fixed distance ruled on a glass plate for every other ten revolutions of the screw from 10 to 280 showed remarkably small periodic errors. At a maximum these amount to 0.3 μ which is much below the limit of accuracy of measurement of spectrum plates. The errors of run of the instrument do not, of course, need to be considered in small differential measurements of this sort. A few of the plates have been measured upon a small comparator built by Gaertner of Chicago. An investigation of the screw of this machine has indicated periodic

errors considerably larger than those of the Toepfer instrument, but they still fall below the errors of measurement and may be neglected.

An important consideration to be borne in mind in the measurement of the plates is that of the inclination of the cross-wire in the eye-piece of the measuring instrument. It is evident that unless this coincides accurately with the inclination of the spectrum lines error will be introduced into the measured displacements, since reversing the plate in the ordinary way does not affect the position of the wire in this regard. The objections to attempting to correct by making the second measurement through the glass are obvious. Accordingly, the following procedure has been followed. A solar spectrum has been photographed with a very long slit, having a horizontal line running through its center due to a fine wire stretched across the slit. This plate is used as a standard, and the vertical wire in the eye-piece of the measuring machine is carefully adjusted until it is accurately parallel to the spectrum lines after the plate has been lined up in the usual way with the aid of the horizontal line. It is evident that with the wire adjusted by the use of these long lines any error in its inclination with reference to the very short lines of the rotation spectra must be quite negligible. After this adjustment has been made the cross-wire of the measuring instrument is clamped in position. A change of position of the grating or any inclination of the slit of the spectrograph would, of course, necessitate a new adjustment of the cross-wire, but this has occurred on only one occasion.

In the conversion of the measured displacements into radial velocity use has been made of a small table which combines into one factor for each line the various reduction factors which it is necessary to employ.

It is of course impossible to give the details of the individual plates within the limits of this article. Accordingly, it has seemed best to include two tables of summaries, the first giving the values of the velocities for each plate derived from a mean of all the lines, and the second the value for each line derived from a mean of all the plates. Both the linear and the angular velocities are reduced to the sidereal period of rotation.

The following table furnishes a summary of the results of the separate plates for each latitude. The values given are the means of all the lines measured.

TABLE I

Number of Plate	Date 1906	Number of Lines	ϕ	v km	Number of Plate	Date 1906	Number of Lines	ϕ	v km
ω 3	May 3	20	9°.9	2.012	ω 25	June 15	20	60°.0	0.848
			24.8	1.803				75.0	0.414
			39.8	1.414	ω 26	June 16	20	0.2	2.088
			54.7	0.995				15.0	1.973
			69.6	0.584				30.0	1.636
			83.6	0.169				44.9	1.271
ω 6	May 8	19	10.7	2.026				59.9	0.852
			25.7	1.789				74.9	0.442
			40.6	1.390	ω 27	June 16	20	0.0	2.077
			55.6	0.981				15.0	1.959
			70.5	0.575				30.0	1.656
			84.6	0.146				45.0	1.263
ω 8	May 19	19	0.8	2.063				60.0	0.862
			15.6	1.969				74.9	0.452
			30.6	1.688	ω 30	Oct. 19	20	0.0	2.109
			45.5	1.317				14.9	1.966
			59.2	0.944				29.8	1.695
			75.4	0.433				44.8	1.293
ω 19	June 12	20	0.0	2.063				59.6	0.876
			15.0	1.946				74.1	0.488
			30.0	1.673	ω 31	Oct. 19	20	0.0	2.110
			45.0	1.271				15.0	1.974
			60.0	0.862				29.9	1.698
			75.0	0.446				44.9	1.312
ω 20	June 12	20	0.0	2.071				59.8	0.898
			15.0	1.932				74.2	0.498
			30.0	1.659	ω 35	Nov. 11	20	0.5	2.056
			45.0	1.262				74.2	0.467
			60.0	0.856	ω 36	Nov. 11	20	0.5	2.078
			75.0	0.439				14.4	1.977
ω 21	June 12	20	0.0	2.060				29.4	1.683
			15.0	1.939				44.4	1.277
			16.0	1.939				59.3	0.889
			30.0	1.664			19	74.2	0.488
			45.0	1.267	ω 37	Nov. 11	20	0.5	2.082
			60.0	0.849				14.4	1.975
			75.0	0.444				29.4	1.689
ω 23	June 15	20	0.1	2.056				44.4	1.276
			15.1	1.937				59.3	0.881
			30.1	1.667				74.2	0.472
			45.1	1.252	ω 38	Nov. 11	20	0.5	2.077
			60.1	0.845				14.4	1.958
			75.0	0.430				29.4	1.670
ω 24	June 15	20	0.0	2.071				44.4	1.273
			15.0	1.939				59.3	0.871
			30.0	1.672				74.2	0.470
			45.0	1.265	ω 39	Nov. 11	20	0.5	2.081
			60.0	0.858				14.4	1.956
			74.9	0.440				29.4	1.677
ω 25	June 15	20	0.0	2.067				44.4	1.281
			15.0	1.961				59.3	0.882
			30.0	1.656				74.2	0.473
			45.0	1.269	ω 39½	Dec. 18	20	1.2	2.099

TABLE I—*Continued*

Number of Plate	Date 1906–7	Number of Lines	ϕ	v km
ω 39½	Dec. 18	20	15°.2	1.969
			30.2	1.696
ω 40	Dec. 18	20	0.2	2.085
			0.2	2.095
			15.2	1.962
			15.2	1.962
			30.2	1.685
			30.2	1.689
ω 41	Dec. 18	20	0.2	2.087
			0.2	2.073
			15.2	1.950
			15.2	1.952
			30.2	1.691
			30.2	1.679
ω 46	Dec. 18	20	44.4	1.285
			44.4	1.282
			44.4	1.282
			59.4	0.877
			59.4	0.868
			59.4	0.871
ω 48	Dec. 18	20	35.4	1.532
			35.4	1.509
			44.4	1.294
			51.9	1.071
			51.9	1.076
			59.4	0.881
ω 50	1907 Feb. 3	20	7.1	2.009
			23.0	1.828
			37.9	1.510
			53.7	1.002
			69.2	0.596
			77.5	0.325
ω 55	Feb. 15	20	7.4	2.046
			22.3	1.838
			38.2	1.458
ω 56	Feb. 15	20	7.4	2.010
			22.3	1.846
			38.2	1.455
			53.9	1.045
			69.4	0.608
ω 60	Feb. 28	20	6.9	2.041
			6.9	2.045
			20.8	1.837
			28.4	1.676
			35.3	1.510
		20	43.6	1.295
			50.7	1.088
ω 61	Feb. 28	18	59.8	0.831
		17	65.6	0.676
		18	65.6	0.676
		17	59.9	0.824
		18	50.9	1.102

Number of Plate	Date 1907	Number of Lines	ϕ	v km
ω 61	Feb. 28	16	44°.0	1.261
ω 62	Feb. 28	20	6.0	2.041
			7.9	1.995
			15.6	1.944
			22.5	1.786
			30.2	1.652
			38.1	1.510
ω 63	Feb. 28	20	7.2	2.035
			20.6	1.841
			28.2	1.672
			35.1	1.533
			50.7	1.055
			43.8	1.280
ω 64	April 7	20	77.5	0.359
ω 67	April 7	20	77.5	0.360
ω 68	April 7	20	77.5	0.365
ω 69	April 7	20	77.5	0.365
ω 81	April 22	20	67.2	0.642
			67.2	0.635
			72.5	0.485
			72.5	0.483
			79.5	0.326
			79.5	0.321
ω 83	May 10	20	63.5	0.749
			63.5	0.747
			74.4	0.441
			74.4	0.437
			79.2	0.311
			79.2	0.303
ω 85	May 30	20	63.8	0.723
			63.8	0.728
			74.8	0.442
			74.8	0.441
			59.8	0.304
			79.8	0.306
ω 86	May 31	20	14.8	1.967
			29.8	1.663
		19	44.8	1.298
		20	64.1	0.750
			76.1	0.392
			81.1	0.267
ω 87	June 22	20	8.1	2.024
			23.1	1.794
			38.6	1.407
			52.1	1.053
			52.1	1.048
			59.1	0.857
ω 88	June 22	20	8.1	2.036
			23.1	1.783
			38.6	1.406
			52.1	1.063
			52.1	1.062
			59.1	0.851

TABLE I—*Continued*

Number of Plate	Date 1907	Number of Lines	ϕ	v km	Number of Plate	Date 1907	Number of Lines	ϕ	v km
ω 89	June 22	20	8°.5	1.990	ω 90	June 22	20	35°.4	1.440
			23.5	1.787				54.5	1.006
			39.0	1.400				53.0	1.063
			52.5	1.077				64.7	0.721
			52.5	1.069	ω 91	June 23	20	6.9	2.011
			59.5	0.856				21.9	1.780
ω 90	June 22	20	6.9	2.044				37.4	1.423
			21.9	1.791				53.0	1.064

The results given in this table have been grouped into mean positions for twelve latitudes, and a summary of the values for these latitudes is found in the latter part of the discussion. In Table II, immediately following, the results are given for the individual lines of the list, the number of plates included under each mean latitude being indicated in the third column of each table. As usual ξ is used to denote angular velocity.

An examination of Table II will lead to several interesting conclusions. The most striking of these is that the two lines due to carbon at λ 4197.26 and 4216.14, and the line due to lanthanum at λ 4196.70, show systematically low values of the angular velocity. The following brief summary indicates more clearly their behavior in this respect, the quantities given being the differences between their values and the mean values for the list.

ϕ	0°.2	7°.7	15°.0	22°.7	29°.7	37°.7	44°.7	52°.7	59°.6	65°.7	74°.9	80°.4
4196.70	0°.0	−0°.1	−0°.2	−0°.1	−0°.1	−0°.1	−0°.2	−0°.3	−0°.4	−0°.3	−0°.7	−1°.1
4197.26	−0.1	0.0	−0.1	−0.1	0.0	−0.2	−0.2	−0.2	−0.2	−0.4	−0.5	−0.9
4216.14	−0.2	0.0	−0.1	−0.1	−0.2	−0.1	−0.2	−0.2	−0.3	−0.3	−0.5	−0.7

In the higher latitudes, of course, a small difference in linear velocity corresponds to a large difference in angular velocity, and the quantitative results are relatively much less certain than in the lower latitudes. Accordingly, while the apparent increase in the size of the differences seems to be marked in the higher latitudes, I do not feel justified at present in concluding that this is an indication that the lower parts of the reversing layer (at which these lines undoubtedly originate for the most part) show a greater retardation

TABLE II

ϕ	λ	Number of Plates	v km	ξ	ϕ	λ	Number of Plates	v km	ξ
0°.2	4196.699	21	2.076	14°.74	15°.0	4284.838	23	1.954	14°.36
	4197.257	21	2.070	14.70		4287.566	23	1.958	14.39
	4203.730	20	2.094	14.87		4288.310	23	1.943	14.28
	4209.144	21	2.105	14.95		4290.377	23	1.935	14.22
	4216.136	21	2.057	14.61		4290.542	23	1.954	14.36
	4220.509	21	2.094	14.87		4291.630	23	1.956	14.38
	4232.887	21	2.082	14.79		4294.936	23	1.947	14.32
	4257.815	21	2.092	14.85	22°.7	4196.699	12	1.791	13.78
	4258.477	21	2.080	14.77		4197.257	13	1.796	13.82
	4265.418	21	2.077	14.75		4203.730	13	1.810	13.93
	4266.081	21	2.085	14.81		4209.144	13	1.818	13.99
	4268.915	21	2.074	14.73		4216.136	13	1.791	13.78
	4276.836	21	2.081	14.78		4220.509	13	1.816	13.97
	4284.838	21	2.073	14.72		4232.887	13	1.814	13.95
	4287.566	21	2.072	14.71		4257.815	13	1.823	14.03
	4288.310	21	2.077	14.75		4258.477	13	1.806	13.90
	4290.377	21	2.062	14.65		4265.418	13	1.801	13.86
	4290.542	21	2.071	14.71		4266.081	13	1.823	14.03
	4291.630	21	2.066	14.67		4268.915	13	1.809	13.92
	4294.936	21	2.069	14.69		4276.836	13	1.803	13.88
7°.7	4196.699	14	2.016	14.44		4284.838	13	1.807	13.91
	4197.257	15	2.026	14.51		4287.566	13	1.806	13.91
	4203.730	15	2.049	14.68		4288.310	13	1.801	13.86
	4209.144	15	2.048	14.67		4290.377	13	1.796	13.83
	4216.136	15	2.028	14.53		4290.542	13	1.804	13.88
	4220.509	15	2.032	14.56		4291.630	13	1.799	13.84
	4232.887	15	2.038	14.59		4294.936	13	1.795	13.81
	4257.815	15	2.054	14.72	29°.7	4196.699	24	1.656	13.54
	4258.477	15	2.031	14.55		4197.257	24	1.668	13.64
	4265.418	15	2.028	14.53		4203.730	23	1.686	13.78
	4266.081	15	2.045	14.64		4209.144	24	1.686	13.78
	4268.915	15	2.029	14.54		4216.136	24	1.648	13.46
	4276.836	15	2.028	14.53		4220.509	24	1.685	13.78
	4284.838	15	2.022	14.48		4232.887	24	1.685	13.78
	4287.566	15	2.017	14.45		4257.815	24	1.692	13.84
	4288.310	15	2.017	14.45		4258.477	24	1.681	13.74
	4290.377	15	2.003	14.34		4265.418	24	1.673	13.68
	4290.542	15	2.007	14.45		4266.081	24	1.683	13.76
	4291.630	15	2.010	14.40		4268.915	24	1.668	13.64
	4294.936	15	2.012	14.41		4276.836	24	1.674	13.68
15°.0	4196.699	23	1.938	14.24		4284.838	24	1.673	13.68
	4197.257	23	1.952	14.35		4287.566	24	1.670	13.66
	4203.730	23	1.974	14.52		4288.310	24	1.669	13.64
	4209.144	23	1.975	14.52		4290.377	24	1.663	13.60
	4216.136	23	1.944	14.29		4290.542	24	1.670	13.66
	4220.509	23	1.979	14.56		4291.630	24	1.674	13.68
	4232.887	23	1.968	14.48		4294.936	24	1.669	13.64
	4257.815	23	1.980	14.57	37°.7	4196.699	15	1.442	12.93
	4258.477	23	1.961	14.41		4197.257	16	1.438	12.90
	4265.418	23	1.966	14.46		4203.730	16	1.462	13.11
	4266.081	23	1.964	14.44		4209.144	16	1.461	13.10
	4268.915	23	1.965	14.45		4216.136	16	1.446	12.96
	4276.836	23	1.958	14.39		4220.509	16	1.459	13.08

TABLE II—*Continued*

ϕ	λ	Number of Plates	v km	ξ	ϕ	λ	Number of Plates	v km	ξ
37°7	4232.887	16	1.461	13°10	52°7	4294.936	18	1.061	12°42
	4257.815	16	1.472	13.20	59°6	4196.699	21	0.831	11.67
	4258.477	16	1.454	13.04		4197.257	21	0.843	11.84
	4265.418	16	1.458	13.07		4203.730	23	0.858	12.05
	4266.081	16	1.465	13.14		4209.144	23	0.869	12.20
	4268.915	16	1.463	13.12		4216.136	23	0.840	11.79
	4276.836	16	1.459	13.08		4220.509	23	0.860	12.07
	4284.838	16	1.455	13.05		4232.887	23	0.865	12.14
	4287.566	16	1.454	13.04		4257.815	22	0.884	12.41
	4288.310	16	1.457	13.06		4258.477	23	0.870	12.21
	4290.377	16	1.446	12.96		4265.418	23	0.869	12.20
	4290.542	16	1.454	13.04		4266.081	23	0.879	12.34
	4291.630	16	1.452	13.02		4268.915	23	0.861	12.09
	4294.936	16	1.457	13 06		4276.836	23	0.866	12.16
44°7	4196.699	21	1.256	12.54		4284.838	23	0.858	12.05
	4197.257	21	1.263	12.62		4287.566	23	0.869	12.20
	4203.730	21	1.275	12.74		4288.310	23	0.864	12.13
	4209.144	22	1.296	12.95		4290.377	23	0.854	11.99
	4216.136	22	1.259	12.58		4290.542	23	0.852	11.96
	4220.509	22	1.287	12.86		4291.630	23	0.853	11.98
	4232.887	22	1.290	12.88		4294.936	23	0.863	12.12
	4251.815	22	1.299	12.96	65°7	4196.699	15	0.676	11.65
	4258.477	22	1.280	12.78		4197.257	16	0.671	11.56
	4265.418	21	1.285	12.84		4203.730	18	0.695	11.98
	4266.081	22	1.289	12.87		4209.144	18	0.698	12.03
	4268.915	21	1.282	12.78		4216.136	18	0.673	11.60
	4276.836	22	1.285	12.84		4220.509	18	0.692	11.93
	4284.838	22	1.280	12.78		4232.887	18	0.698	12.03
	4287.566	22	1.278	12.76		4257.815	18	0.710	12.24
	4288.310	22	1.271	12.70		4258.477	18	0.694	11.96
	4290.377	22	1.271	12.70		4265.418	18	0.696	11.99
	4290.542	22	1.274	12.72		4266.081	18	0.712	12.27
	4291.630	22	1.274	12.72		4268.915	18	0.697	12.01
	4294.936	22	1.282	12.80		4276.836	18	0.692	11.93
52°7	4196.699	16	1.030	12.06		4284.838	18	0.692	11.93
	4197.257	17	1.036	12.13		4287.566	18	0.694	11.96
	4203.730	18	1.049	12.28		4288.310	18	0.689	11.87
	4209.144	18	1.048	12.27		4290.377	18	0.690	11.89
	4216.136	18	1.036	12.13		4290.542	18	0.694	11.96
	4220.509	18	1.052	12.32		4291.630	18	0.698	12.03
	4232.887	18	1.050	12.30		4294.936	18	0.697	12.01
	4257.815	18	1.065	12.47	74°9	4196.699	37	0.409	11.16
	4258.477	18	1.055	12.35		4197.257	37	0.415	11.32
	4265.418	18	1.053	12.33		4203.730	36	0.436	11.90
	4266.081	18	1.069	12.52		4209.144	37	0.440	12.01
	4268.915	18	1.052	12.31		4216.136	37	0.416	11.35
	4276.836	18	1.054	12.34		4220.509	37	0.436	11.90
	4284.838	18	1.058	12.38		4232.887	37	0.436	11.90
	4287.566	18	1.055	12.35		4257.815	37	0.449	12.25
	4288.310	18	1.060	12.41		4258.477	37	0.435	11.87
	4290.377	18	1.052	12.31		4265.418	37	0.437	11.92
	4290.542	18	1.057	12.37		4266.081	37	0.449	12.25
	4291.630	18	1.057	12.37		4268.915	37	0.436	11.90

TABLE II—*Continued*

ϕ	λ	Number of Plates	v km	ξ	ϕ	λ	Number of Plates	v km	ξ
74°9	4276.836	37	0.439	11°98	80°4	4232.887	11	0.292	12.48
	4284.838	37	0.434	11.84		4257.815	11	0.290	12.39
	4287.566	37	0.442	12.06		4258.477	11	0.281	12.01
	4288.310	37	0.439	11.98		4265.418	11	0.287	12.26
	4290.377	37	0.443	12.09		4266.081	11	0.294	12.56
	4290.542	37	0.442	12.06		4268.915	11	0.293	12.52
	4291.630	37	0.443	12.09		4276.836	11	0.283	12.09
	4294.936	36	0.434	11.84		4284.838	11	0.291	12.43
80.4	4196.699	10	0.257	10.98		4287.566	11	0.291	12.43
	4197.257	11	0.250	11.11		4288.310	11	0.285	12.18
	4203.730	11	0.281	12.01		4290.377	11	0.280	11.96
	4209.144	11	0.271	11.58		4290.542	11	0.286	12.22
	4216.136	11	0.267	11.41		4291.630	11	0.293	12.52
	4220.509	11	0.285	12.18		4294.936	11	0.284	12.13

toward the pole than do the higher portions, although some such effect is by no means improbable. In this connection it is interesting to note that the variations in angular velocity found by Halm for the different years covered by his observations were greatest toward the pole.[1] As to the reality of the differences in the rotation rate as given by these lines, and by the mean of the entire list, there can, however, be no question, and the inference is justified that the vapors giving rise to these lines have an angular velocity of rotation which is less than the average rate of the reversing layer. In the lower latitudes the difference amounts to about 0°1 in the daily rate, which would mean a difference of about four hours in the equatorial period of rotation. For both of these elements we have independent evidence tending to show that they lie at a low level in the sun's atmosphere. In the case of carbon this is furnished by direct visual observations, while the great weakening at the limb of the lines of lanthanum and other elements of similarly high atomic weight indicates a comparatively low-lying origin for these elements as well.

Of the other lines in the list the line due to titanium at λ 4290.38 is perhaps the most interesting. This also shows a systematically low value for the angular rotation, although the difference is not so great as in the case of the carbon and lanthanum lines. It is strongly enhanced in the spark, and according to one of the more commonly

[1] *Astronomische Nachrichten*, **173**, 296, 1907.

accepted views of the enhanced lines would lie at a comparatively high level in the sun's atmosphere. It is, however, weakened at the limb, and shows a considerable shift toward the red at the limb as compared with its position at the center.[1] The weakening may perhaps be ascribed to temperature effects, but the pressure-shift and the lower rotational value are strong indications that the line originates in part, at least, at a low level.

The cases of lines giving high rotational values seem to be hardly so marked as those giving the low values which we have just discussed, although the two lines of manganese at λ 4257.82 and 4266.08 give results which are consistently large. The second of these lines is identified by Frost as present in the flash spectrum, and there is a line in his list close to the position of the first as well, although no identification is made.[2] Neither line, however, is conspicuous in intensity. At the sun's limb, beyond a slight widening, the lines seem to be little affected.

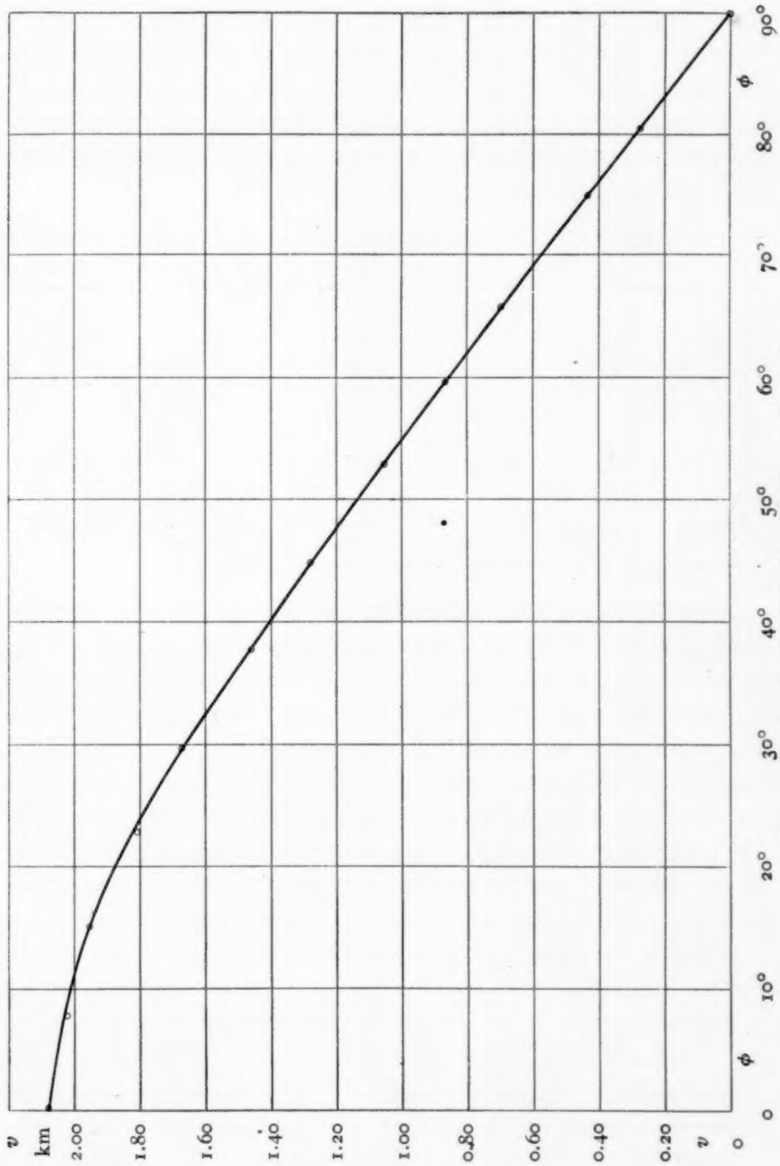
In this connection reference should be made to the work of Jewell at Johns Hopkins University in 1896. While no details of this work have ever been published, some results obtained by him are referred to in an editorial note in the *Astrophysical Journal.*[3] From his investigations Jewell concluded that the outer and inner portions of the sun's atmosphere show a difference in rotation-period amounting to several days, the lower portions having the longer period. The results found here agree with his as regards the direction of the retardation, but it would appear that the amount must be much less than that found by him. Jewell also concluded that at the lower levels the equatorial acceleration is small. So far as we may draw any inference from the result for the carbon and lanthanum lines it would seem to be decidedly opposed to this view. Jewell's conclusion that the carbon lines lie at a very low level is fully confirmed.

After this discussion of the behavior of the individual lines we may return to a consideration of the general results. Although it is clear

[1] A full discussion of this effect, first found by Halm, and later confirmed and extended by Professor Hale and myself, will be published at an early date. Our observations show that it is almost certainly due to pressure, although it may be modified by other causes as well.

[2] *Astrophysical Journal*, **22**, 335, 336, 1900.

[3] *Ibid.*, **4**, 138, 1896.



Curve Showing Radial Velocity at Different Latitudes

that different lines may give different values for the rate of rotation, it would seem that in order to obtain an average value for the rotational velocity of the reversing layer we can hardly do better than to take a general mean for all the lines. If we form mean values from the quantities given in Table I we are led to the following summary. In the formation of the means such plates as have been measured twice have been assigned double weight.

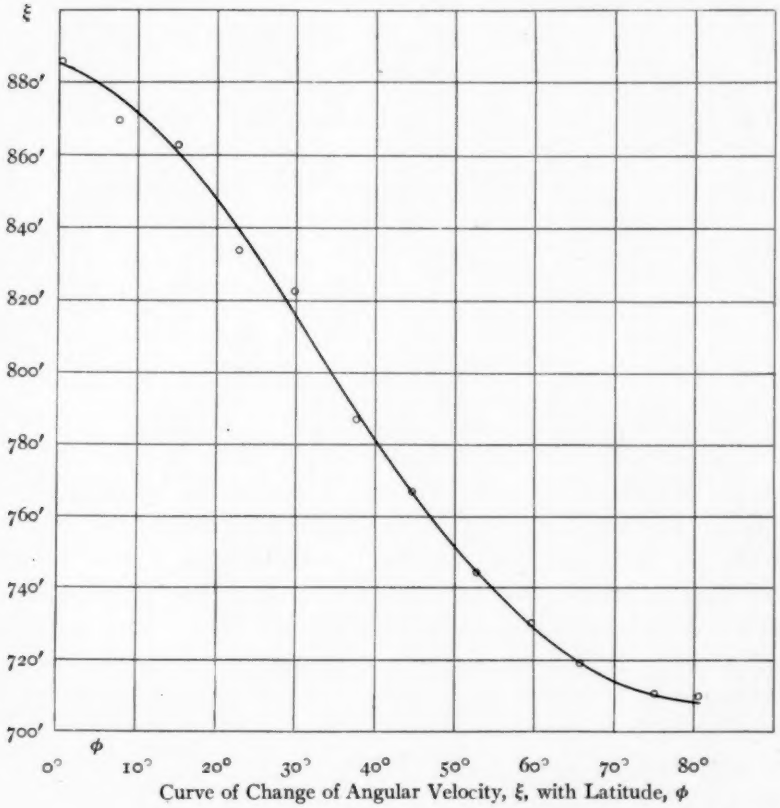
ϕ	Weight	v km	ξ	Period Days
0°2	21	2.078	14°75	24.39
7.7	15	2.023	14.50	24.83
15.0	23	1.957	14.39	25.01
22.7	13	1.808	13.92	25.86
29.7	24	1.673	13.68	26.32
37.7	15	1.461	13.11	27.46
44.7	23	1.279	12.77	28.19
52.7	18	1.055	12.35	29.15
59.6	24	0.864	12.13	29.68
65.7	20	0.696	11.99	30.02
74.9	33	0.434	11.85	30.38
80.4	11	0.277	11.84	30.40

The two curves which accompany this paper give a graphical representation of the quantities in the table above. In the first and larger curve the radial velocities are plotted as ordinates with the latitudes for abscissae. The second curve represents the change of the angular velocity ξ with the latitude. Both of these curves have been drawn with due regard to the weights of the normal points, which accounts for the apparently abnormal deviation from the curves of the points of lower weight. This is especially true for the two points at 7°7 and 22°7, which are based on comparatively few observations, and which show by far the largest deviations from both curves.

One of the most interesting features of these results is the form of the angular velocity-curve. Starting with a curvature strongly convex upward, its slope rapidly becomes very steep. At about 30° or 35° of latitude there is a point of inflection, and in the higher latitudes it approaches the asymptotic form. In other words, the rate of change of the angular velocity of rotation with the latitude increases from the equator to about latitude 30°, at which point it is greatest. It then begins to decrease, and in the highest latitudes becomes very small. An extrapolation from the curve gives for the daily angular rotation

rate at the pole a value of 11°.7, which would correspond to a period of rotation of 30.6 days.

In order to facilitate the comparison of these results with those of Dunér and Halm, a short table is appended giving their values for the latitudes which we have employed here. Their results are taken from



Curve of Change of Angular Velocity, ξ, with Latitude, φ

the papers already referred to.[1] Since Dunér's values are confined to the six latitudes from 0° to 75°, differing by intervals of 15°, his results are given for these latitudes alone. The much greater number of latitudes employed by Halm, however, makes it comparatively

[1] In his paper Halm has derived mean values from his series of observations for 1901 to 1906, although he ascribes the large systematic differences of the results for different years to actual variations in the period of rotation. His mean values are employed here.

simple to construct a curve and take from it with sufficient accuracy the values corresponding to the latitudes required. The quantities given in the table have been obtained in this way, and have, of course, a considerable advantage over those given by Dunér and myself, since they have had the benefit of the smoothing-out effect of the curve.

Linear Velocity				Angular Velocity		
ϕ	Dunér km	Halm km	Adams km	Dunér	Halm	Adams
0°2	2.08	2.05	2.08	14°8	14°6	14°7
7.7		2.02	2.02		14.5	14.5
15.0	1.97	1.95	1.96	14.5	14.3	14.4
22.7		1.83	1.81		14.1	13.9
29.7	1.70	1.68	1.67	13.9	13.7	13.7
37.7		1.49	1.46		13.4	13.1
44.7	1.28	1.32	1.28	12.8	13.2	12.8
52.7		1.10	1.05		12.9	12.4
59.6	0.82	0.90	0.86	11.5	12.6	12.2
65.7		0.72	0.69		12.4	12.0
74.9	0.39	0.45	0.43	10.7	12.3	11.8
80.4		0.29	0.28		12.4	11.8

An inspection of these results shows that in the lower latitudes all three series of observations give values which are fairly accordant. Above 30° of latitude, however, Halm's results become larger than those in the other two series, and this continues to be true in the higher latitudes. At about 45° or 50° Dunér's values cross my own, and fall considerably below in the higher latitudes. The general conclusion accordingly, is that the photographic results give a curve of angular velocities which in the higher latitudes is intermediate between those of Dunér and Halm. This curve agrees with that of Halm in showing a falling-off in the rate of the variation in higher latitudes, but the effect seems to begin at a lower latitude than is indicated by Halm's results.

As regards the interesting question of a long period variation in the rotation rate the results given here are, of course, not decisive, since the interval covered by them amounts to rather less than fourteen months. For this interval there seems to be no variation of appreciable size. The fact, moreover, that the values found agree as well as they do with the mean values of Halm for his entire series of observations would seem to furnish some presumption against the existence of such a variation, at least of such magnitude as was found by him.

At present the question must be regarded as one for future observations to decide.

Since the permanency of form of the velocity-curve is thus open to possible doubt, it has not seemed desirable to devote any large amount of attention at present to the consideration of empirical equations which might satisfy it. A preliminary solution by least squares of an equation of the form given by Faye,

$$v=(a-b \sin^2 \phi) \cos \phi,$$

showed that the curve could be reasonably well satisfied by an equation of this type, the largest residual amounting to about 0.024 km. The residuals (computed−observed values) were, however, consistently positive in mean latitudes, and consistently negative in high latitudes. This naturally suggested the addition of a term in cos ϕ, giving an equation of the form

$$v=(a-b \sin^2 \phi+ c \cos \phi) \cos \phi ,$$

or, in another form,

$$v=(a'+b' \cos \phi+c' \cos^2 \phi) \cos \phi .$$

A solution by least squares of this equation for the twelve latitudes gave the following residuals:

ϕ	C.−O. km
0°.2	−0.008
7.7	+0.016
15.0	0.000
22.7	+0.013
29.7	−0.007
37.7	+0.004
44.7	−0.004
52.7	0.000
59.6	0.000
65.7	−0.001
74.9	+0.002
80.4	+0.004

The only large residuals are those given by the points of low weight at 7°.7 and 22°.7, and these are by no means excessive. Though an equation involving three constants is, of course, inferior to one containing but two, the very satisfactory size of the residuals given by it, and the simplicity of its form probably justify its use.

In concluding this discussion it will be useful for purposes of comparison with the results obtained from the measures of spot, faculae,

and flocculi positions, to add a short table giving the values of the daily angular rotation for every 10° of latitude. These have been taken from the curve and are as follows.

ϕ	ξ	Period Days
0°0	14°72	24.46
10.0	14.52	24.79
20.0	14.13	25.48
30.0	13.62	26.43
40.0	13.03	27.63
50.0	12.53	28.73
60.0	12.15	29.63
70.0	11.90	30.25
80.0	11.78	30.56

A comparison of the probable errors of these results with the probable errors of the visual determinations of Dunér and Halm is somewhat difficult on account of the difference in the character of the measurements. In the work of both Dunér and Halm a considerable number of settings of the micrometer wire were made upon each of two lines (by Dunér twelve to twenty-four, by Halm eight), and these series of settings, combined for the two lines, furnish separate observations of the velocity. In the present photographic investigation a smaller number of settings was made upon each of a considerable number of lines, and the values given by all the lines measured on a plate are combined to form a single determination. For general purposes, however, it will be sufficient to compare the probable error in the determination from a single line on the photographic plate, with the probable error from a series of visual observations equal in number to that of the lines on the plate. This evidently gives a decided advantage to the visual results in the comparison, since the mean of two lines is used for them as well as a greater number of settings on each line. On the other hand it is clear that in the photographic results such lines as give systematically large or small values throughout the whole series of observations should be omitted in the formation of the probable error. We have discussed six cases of this sort in connection with Table II, namely, the lines λ 4196, 4197, 4216, 4257, 4266, and 4290.38. If we omit these we have left a total of fourteen lines to each plate. A determination made from several

plates taken at random from the series gives as the probable error for a single line,

$$\epsilon = \pm 0.015 \text{ km};$$

or, for the mean value from the plate,

$$\epsilon_0 = \pm 0.004 \text{ km}.$$

To compare with these we have a series of determinations by Halm in 1903[1] averaging fifteen observations for each latitude. He gives for these

$$\epsilon = \pm 0.070 \text{ km}$$

as the probable error of a single observation, and

$$\epsilon_0 = \pm 0.018 \text{ km}$$

as the probable error of the group. Dunér has not given the probable errors for his completed series of observations. For his earlier results they amount to about double those given by Halm.

We are certainly justified in concluding from this comparison that for the same number of measurements the photographic method is capable of furnishing results of higher precision than the visual, at least in so far as inferences of this kind can be drawn from comparisons of probable errors. As in most cases of quantitative spectroscopic work, however, it is probable that in both the visual and the photographic series of observations the effects of small systematic errors begin to be felt before the limits of accuracy defined by the probable errors of groups of results are reached. As regards this class of error it is difficult to conclude with which method of observation the advantage lies. Since the observer is free during the exposure of the photographic plate to do any small amount of guiding necessary to hold the image of the sun in a definite position, the error arising from wandering of the image should be less than in visual measures by a single observer. On the other hand any error which does enter from this source affects all of the lines upon the photographic plate, while in the visual measures it affects each set of pointings only. Perhaps the most valuable general conclusion that can be drawn from the discussion is that the degree of accuracy of measurement attainable on the photographs is so high that it warrants the use of the greatest precautions to avoid small systematic errors.

[1] *Transactions of the Royal Society of Edinburgh*, **41**, Part 1, 96.

The investigations will be continued with the use of the more powerful apparatus of the tower telescope, and it is hoped that a substantial gain in accuracy may be attained. Among the superior advantages for such work possessed by this instrument we may mention the following: greater linear scale of the plates; a higher degree of accuracy of setting for the various position angles on the sun's image, as well as the possibility of reaching all latitudes on the sun's surface at all times of the year; less liability to changes of temperature on the part of the grating during the exposures; and finally some improvement in the definition of the solar image, and greater freedom from astigmatism and change of focus while the photographs are being obtained.

The more important conclusions derived from this investigation may be summarized as follows:

1. In lower latitudes the values obtained for the rotational velocity agree closely with those of Dunér and Halm. In higher latitudes they are intermediate between the results of these two observers.

2. The rate of change of the rotational velocity with the latitude is greatest at about 30° of latitude. It becomes less in higher latitudes, and beyond 70° is very slight.

3. Different lines give slightly different rates of rotation. Lines of carbon and lanthanum, elements which lie at a low level in the sun's atmosphere, give values for the daily rate about $0\overset{\circ}{.}1$ less than the mean values for all the lines. An enhanced line of titanium also gives a slightly lower rate of rotation, while two lines of manganese included in the list give systematically high results.

4. There is no appreciable variation in the rate of rotation during the fourteen months covered by the observations.

5. A comparison of probable errors indicates a substantial gain in accuracy for the photographic results as compared with the visual, so far as accidental errors of measurement are concerned.

I am indebted to Professor Hale for an active interest in this research, and many valuable suggestions during its progress; also to Miss Lasby of the Computing Division for her most efficient performance of the exacting work involved in the measurement and reduction of the large number of plates.

MOUNT WILSON, CAL.
September 1907

THE SELECTIVE REFLECTION OF SALTS OF CARBONIC AND OTHER OXYGEN ACIDS[1]

By LEIGHTON B. MORSE

I. THE SELECTIVE REFLECTION OF CARBONATES AS A FUNCTION OF THE ATOMIC WEIGHT OF THE BASE

If there were a regular displacement of the resonance periods of simple molecules in salts having the same acid radical, it should first be sought in compounds of simple bases having the same valence. The carbonates seemed especially well adapted to such a study since a large number could be obtained in a suitable form as minerals. The absence of water of crystallization adds further to the simplicity of the molecular structure of the carbonates of the eight elements (*Mg, Ca, Mn, Fe, Zn, Sr, Ba,* and *Pb*) examined, for which RCO_3 may be written as a general formula, R being a bivalent metal.

Partial data were obtainable on the reflection of the carbonates of two elements, calcium and magnesium. E. Aschkinass,[2] in a study of anomalous dispersion, had recorded the reflection of calcite and marble. He found maxima in the reflection of calcite at 6.67 μ and 11.40 μ, and of marble at 6.69 μ and 11.41 μ. By the method of "Reststrahlen" as discovered by H. Rubens and E. F. Nichols,[3] J. T. Porter[4] found a maximum in the reflection of white marble at 6.77 μ.

Some time later W. W. Coblentz[5] gave the reflection of calcite from 4 μ to 11 μ, and of magnesite from 4 μ to 12.4 μ, missing a second band in magnesite and not continuing to the second band in calcite, or to the region of the third bands in either. In the work of Aschkinass, Coblentz, and the present writer the wave-length determinations were referred to the dispersion of a rock-salt prism.

[1] *Phoenix Physical Laboratory Contributions, No. 11.*

[2] *Annalen der Physik,* **1**, 60, 1900.

[3] *Ibid.*, **60**, 418, 428, 1897.

[4] *Astrophysical Journal,* **22**, 229, 1905.

[5] *Investigations of Infra-red Spectra,* Parts III and IV, 81, 1906.

ARRANGEMENT OF APPARATUS AND ADJUSTMENTS

When the shutter at *K* was raised the image of a Nernst glower at *N* (Fig. 1)[1] was focused by the silvered concave mirror M_1 upon the surface under examination at *S*. A second silvered concave mirror M_2 caught the reflected beam and formed a secondary image of the source on the collimator slit *C*. These mirrors, M_1 and M_2, were adjusted near each other in order to make the angle of incidence on the surface at *S* as small as possible.

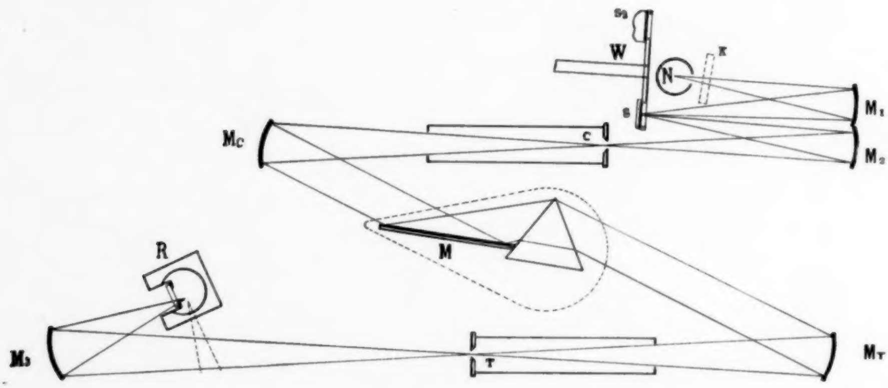


FIG. 1

After resolution by the spectrometer, the section of the spectrum passing through the rear slit *T* was focused on one of the radiometer vanes by a similar silvered concave mirror M_3. The mirrors M_1 and M_2 were adjustable about horizontal and vertical axes perpendicular to the axes of the mirrors. A base provided with leveling screws aided in the adjustment of the mirror M_3.

A small angle of incidence on the surface whose reflection was to be measured, 5° to 6°, was one of the advantages of this arrangement. Also the ability to use plane surfaces of small dimensions, but a little larger than the Nernst glower, made it less difficult to obtain suitable specimens.

Source.—A Nernst glower, operated on alternating current from the city lighting circuit was first used, but irregular fluctuations of

[1] The writer wishes to thank Mr. C. C. Chapin of the department, for valuable assistance given in preparing the curves and for drawing the figure.

the voltage produced not only sudden changes in the intensity of the radiation, but the expansion and contraction of the end wires continually shifted the image of the glower on the slit. Later a direct current glower (0.8 ampere, 110 volts) was used in a 120-volt storage-battery circuit, and slow variations in the current could be compensated for by a variable resistance, with the aid of an ammeter in circuit, reading to hundredths of an ampere.

After making these changes and improving the asbestos chimney used to protect the glower from variable air currents, conditions were so constant that no difficulty was experienced in holding the image of the glower on the spectrometer slit, *C*, for hours with no apparent shift in its position, and the emission of the glower remained equally constant. But for its selective emission, the Nernst glower used in this way would have been an ideal source.

Mounting of the specimens.—Three of the plane-polished mineral surfaces for which the reflection was to be measured, and a polished plane silver mirror were mounted over the four holes 2 cm in diameter in a wheel *W* (Fig. 1). The back face of the disk against which the surfaces were laid was ground as plane as possible on plate glass. But the final adjustment of the surfaces, to bring them successively into the same position when the wheel was rotated from one position to the next, was made by observing the reflected image of an incandescent-lamp filament in the field of a telescope with cross hairs. The smaller surfaces were mounted in cork and then adjusted on the wheel.

Spectrometer.—A Schmidt and Haensch reflection spectrometer with mirrors of 4 cm aperture and 35 cm focal length was used. By the aid of the Wadsworth[1] mirror-prism arrangement, the spectrometer arms remained fixed; and adjustment for the minimum deviation of one wave-length held for all. The face of the rock-salt prism was 5 cm by 8 cm and its refracting angle 59° 59′ 15″. A rotation of the spectrometer arm by 4′ 50″ moved the center of the slit 1 mm. As the front and rear slits were always of equal width, the purity of the spectrum at any setting was as great as the energy necessary for a sufficient radiometer deflection would allow.

Wave-lengths were calculated from the indices of refraction for

[1] F. L. O. Wadsworth, *Phil. Mag.*, **38**, 337, 1894.

rock salt given by H. Rubens[1] to 8.67 μ and the corrected values of H. Rubens and A. Trowbridge[2] were used for longer waves. Rotating the prism-table carrying the Wadsworth mirror-prism arrangement 1′, corresponded to a change in wave-length from 6.50 μ to 6.60 μ, from 11.60 μ to 11.65 μ, or from 14.20 μ to 14.24 μ, respectively.

As the spectrometer was arranged with its arms parallel and close to the prism-table, it was impossible to use any circular hood to protect the salt prism from moisture when it was not in use. In order that greater care might be taken to avoid touching the prism-table when removing and replacing the box used for this purpose, its top was made of glass. The plane sides and semi-cylindrical end of copper, outlined by dotted line in Fig. 1, left a space of 1 cm between the box and the pointed glass prism-table. A brass plate was mounted below the prism-table on a heavy collar about the spectrometer axis. The lower edge of the box fitted into a trough about the outer edge of this plate. Glycerine was used in the trough as a seal and P_2O_5 served as a drying agent.

Radiometer.—Behind the mirror M_3 was an inner brick wall to which the heavy shelf supporting the radiometer R and mirror M_3 (Fig. 1) was fastened. The radiometer was inclosed by a blackened compoboard box, not shown in the diagram. The Nichols' radiometer used was so similar to those described by other observers that only a few details of its construction are necessary. The mica vanes, 0.75 mm by 5 mm, were mounted about 5 mm apart and their front surfaces blackened with platinum black, held by shellac. The window of rock salt was protected by a P_2O_5 dryer when the radiometer was not in use. The half-period of the suspended system varied from twenty-five to fifty-five seconds, depending upon the air pressure in the radiometer. Often the general form of a reflection curve was obtained on one day and observations requiring the most sensitive conditions were made the following day, when the leak had increased the pressure, and with it the period and sensibility of the radiometer. The leak was so small that sufficiently accurate measurements have been made on the third day after the radiometer was pumped out, but the longer

[1] *Annalen der Physik*, **54**, 482, 1895.

[2] *Ibid.*, **60**, 733, 1897.

period made observing tedious. Generally the pressure used was such that the radiometer was slightly ballistic.

Because of the symmetry in the construction of the radiometer suspension, tremors of the building due to the heavy traffic outside at no time interfered with the progress of the work. Throughout the work no trouble arising from static charges on the radiometer vanes was encountered, not even when the stop-cock connecting the radiometer with the mercury pump was left open, a condition mentioned by Coblentz[1] as especially favorable for static disturbances. This was doubtless due to the presence of a small amount of radioactive material placed in the bottom of the radiometer case.

The image of an incandescent-lamp filament, projected on a scale one meter distant by a light concave mirror[2] attached to the lower end of the radiometer suspension, served to indicate deflections. An asbestos box covered the lamp used as an index and permitted light to pass from it only through a narrow slit on the side toward the radiometer. Diaphragms were used in addition to protect the radiometer from the heat of the lamp.

The walls of the room, an inner grating room, were light-tight with the exception of protected openings for ventilation. Unslaked lime was always kept in trays about the room to reduce the moisture in the air.

METHOD OF OBSERVING

A zero reading was taken before and after each deflection. Conditions were held so constant that the mean of two observations on the silver surface, one made before, and the other after observations on the three mineral surfaces, was sufficiently accurate for most of the work. There have been differences in these two deflections from silver which would have caused errors greater than those in reading the deflections from the mineral surfaces. Such occurrences were rare, even at points in the spectrum where the reflection from the mineral surfaces was high, and such observations were invariably repeated. But when the greatest accuracy in determining the posi-

[1] *Investigations of Infra-red Spectra*, Part I, Appendix III.

[2] To Dr. S. R. Williams the writer is much indebted for plating the mirror with platinum. The reflecting surface obtained remained in good condition throughout the work.

tion of a maximum was required, comparison was made between the mineral and silver surfaces at equi-distant points over the crown of the curve. These observations were then repeated with new settings on the same series of points in the spectrum and the maximum was determined from the average results.

When preliminary observations indicated little difference in the reflection maxima of two substances, as for example calcite and aragonite, both $CaCO_3$, or siderite, and rhodochrosite ($Fe = 55.5$, $Mn = 54.6$), they were mounted in the wheel together with the silver mirror. This made it possible to determine the reflection of the two under almost identical conditions and slight errors in reading the spectrometer vernier were eliminated from the comparison. When the position found for a maximum seemed irregular, as for example the band of smithsonite in the third region, in addition to the usual frequent checking of the calibration curve of the spectrometer by setting on the sodium line, further assurance was sought by repeating observations with the substances in question and calcite in the "wheel" together.

DESCRIPTION AND ANALYSES OF SPECIMENS

Three of the mineral reflecting surfaces were polished by Schmidt and Haensch, witherite, strontianite, and aragonite. The others were polished in the laboratory. The polish required for such long waves was much less than for ordinary optical measurements. In fact the magnesite specimens containing silica were used when at normal incidence the image of an incandescent lamp one meter distant was barely visible in a reading-telescope. The range of quality of surfaces was from brilliant to very dull in the order given: witherite, strontianite, smithsonite, aragonite, calcite, rhodochrosite, siderite, cerussite, magnesite.

The writer is much indebted to Dean William Hallock for a number of useful suggestions concerning the preparation of the specimens and also for his friendly interest shown in many other ways.

To Professors Moses and Luquer and to Mr. Lamme, of the Department of Mineralogy, he is variously indebted for kindly advice in mineralogical matters and for the generous loan of materials from the departmental collection.

He is further indebted to Professor L. P. Gratacap, of the Museum

of Natural History, for the following description of the specimens used. The analyses were made by Dr. H. T. Beans, of the university.

1. Magnesite, No. 2, $MgCO_3$, from Oberdorf, Styria. Section from a mass, subcrystalline, fibrous, white; slight schillerization on surface from crystalline texture. Contained magnesium calculated as MgO, 15.94 %; calcium calculated as CaO, 31.73%; Silica, 1.03%; water liberated at bright red heat, 0.48%.

2. Calcite, $CaCO_3$. Polished rhombohedral cleavage surface. Perfect texture, transparent.

3. Aragonite, $CaCO_3$. Section of transparent crystal. Surface used was a polished prism face.

4. Rhodochrosite, $MnCO_3$, from John Reed Mine, Lake Co., Col. Pale pink, semi-transparent, surface nearly parallel to face of rhombohedron.

5. Siderite, $FeCO_3$. Light brown, crystalline, lamellar. Contained iron calculated as FeO, 44.54%. Qualitative tests show some ferric iron and large quantities of manganese. No other qualitative tests were made.

6. Smithsonite, No. 1, $ZnCO_3$. Crystalline, dense texture, marked by lines of spheroidal intergrowth, pale yellow, to mottled in color, transparent, darkened by iron oxide nodule. Contained zinc calculated as ZnO, 60.58%; water liberated at bright red heat, 0.67%. The sample contains considerable iron.

7. Smithsonite, No. 2, $ZnCO_3$, from Laurium, Greece. Compact, fibrous, crystalline, pale milky gray in color, sub-transparent; original surface slightly mammilated. Contained zinc calculated at ZnO, 61.83%; water liberated at bright red heat, 0.59%.

8. Strontianite, $SrCO_3$. Massive, sub-fibrous, distinctly crystalline in structure, pale asparagus green.

9. Witherite, $BaCO_3$. Massive, fibrous, columnar in structure, pearl gray.

10. Cerussite, $PbCO_3$, from Monte Poni, Sardinia. Section obtained from twinned crystal, white, transparent.

RESULTS

Three bands of marked reflection[1] were found in all of the specimens examined (Figs. 2–7), but in no case were there more than three bands found in the spectrum between 4 μ and 15 μ. The bands fall into three distinct regions in the spectrum grouped about 6.7 μ, 11.5 μ, and 14.5 μ. To this grouping only one exception was found, magnesite, No. 1; and from the shape of its reflective curve the presence of a silicate was suspected. This was verified by a chemical analysis, which showed 8.95 per cent. of silica, calculated

[1] The reflection percentages given in the curves are based upon silver assumed total. The actual reflection of silver given by E. Hagen and H. Rubens (*Annalen der Physik*, **11**, 73, 1903) increases from about 98 per cent. at 4μ to 99 per cent. at 14μ.

as SiO_2, together with 4.56 per cent. of water liberated at bright-red heat. Because of these impurities this specimen will not be considered further.

These regions where the carbonate reflection bands occur are distinct from the regions where the salts of other acids as far as known show reflection maxima. This verifies and gives a broader foundation for the conclusion reached by A. H. Pfund[1] based on a study of single bands in several nitrates and sulphates: "That the mechanism giving rise to these maxima was localized within the acid radical."

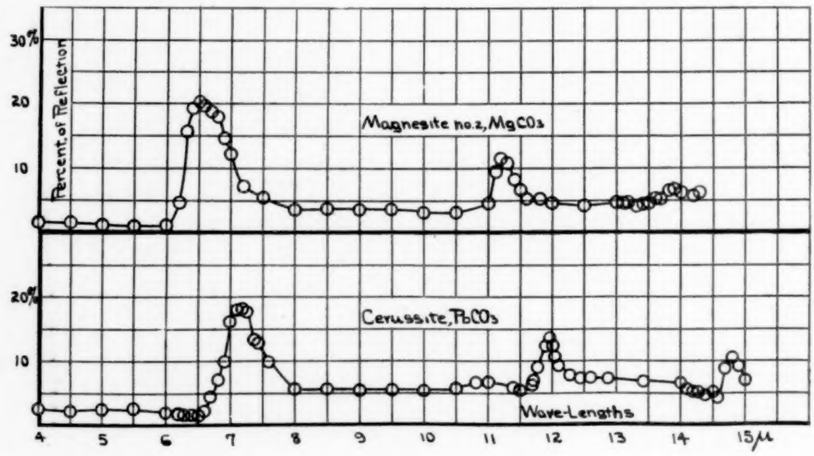


FIG. 2

In general the larger the atomic weight of the base the greater was the wave-length where the maximum reflection occurred, as will be seen by an examination of Figs. 2–7, of which Fig. 8 presents a condensed summary.

The following exceptions appear in the reflection curves found: siderite ($FeCO_3$) in the first region, either smithsonite ($ZnCO_3$) or siderite ($FeCO_3$) and rhodochrosite ($MnCO_3$) in the second region, and all three in the third region. Also, the reflection maxima of calcite and aragonite, both $CaCO_3$, differ considerably in the second region both in magnitude and position; and the aragonite maximum may lie a little toward the long waves from calcite in the first region.

[1] *Astrophysical Journal*, **24**, 23, 1906.

In attempting to answer the question: Can a law be found for the general shifting of the bands toward the long waves with the increase

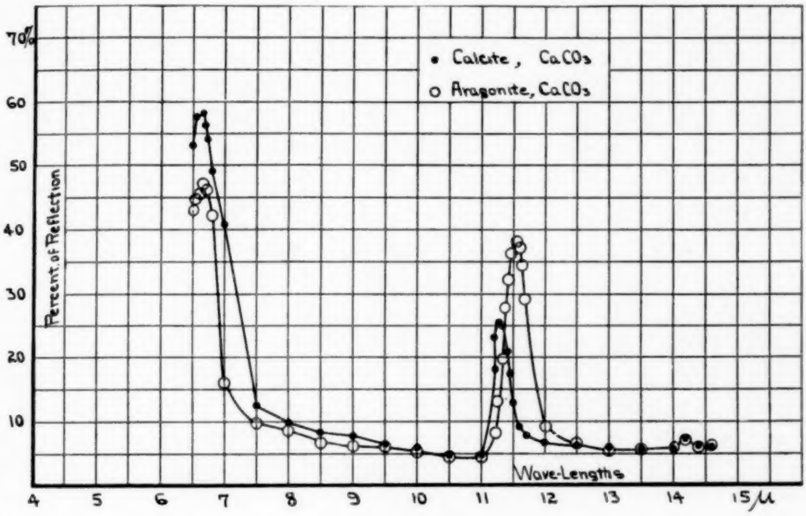


FIG. 3

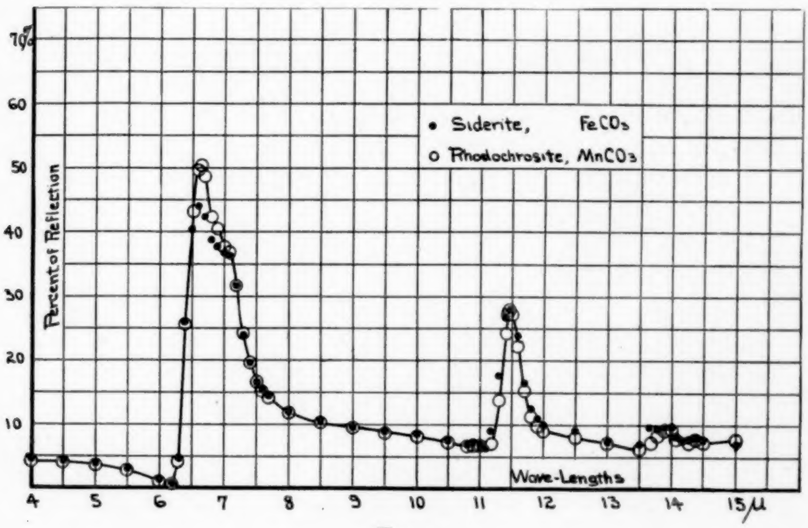
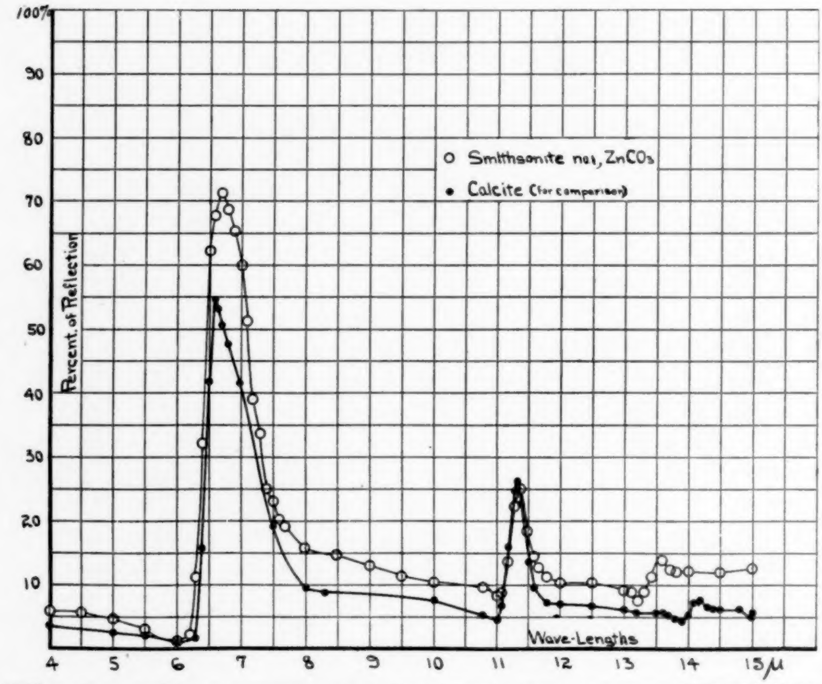


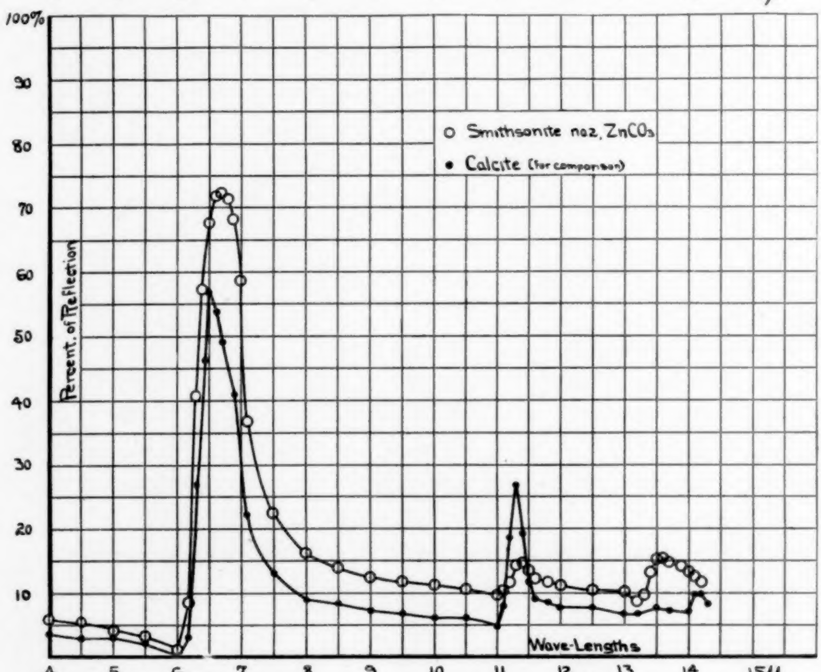
FIG. 4

in atomic weight of the base? a straight line was drawn in each region through the cerussite ($PbCO_3$) and calcite ($CaCO_3$) maxima. These



100%
90
80
70
60
50
40
30
20
10
Percent. of Reflection
o Smithsonite no1, ZnCO3
• Calcite (for comparison)
Wave-Lengths
4
5
6
7
8
9
10
11
12
13
14
15μ

FIG. 5



100%
90
80
70
60
50
40
30
20
10
Percent. of Reflection
o Smithsonite no2, ZnCO3
• Calcite (for comparison)
Wave-Lengths
4
5
6
7
8
9
10
11
12
13
14
15μ

FIG. 6

points were selected to determine the lines $C1$, $C2$, $C3$, Fig. 8, because of the large difference in the atomic weight of the bases. Calcite rather than magnesite was selected as the lower point in determining the line because its structure gave evidence of its being the purer specimen.

Straight lines fitted the results better than any simple curve, showing that in general the shifts in position of the maxima are directly

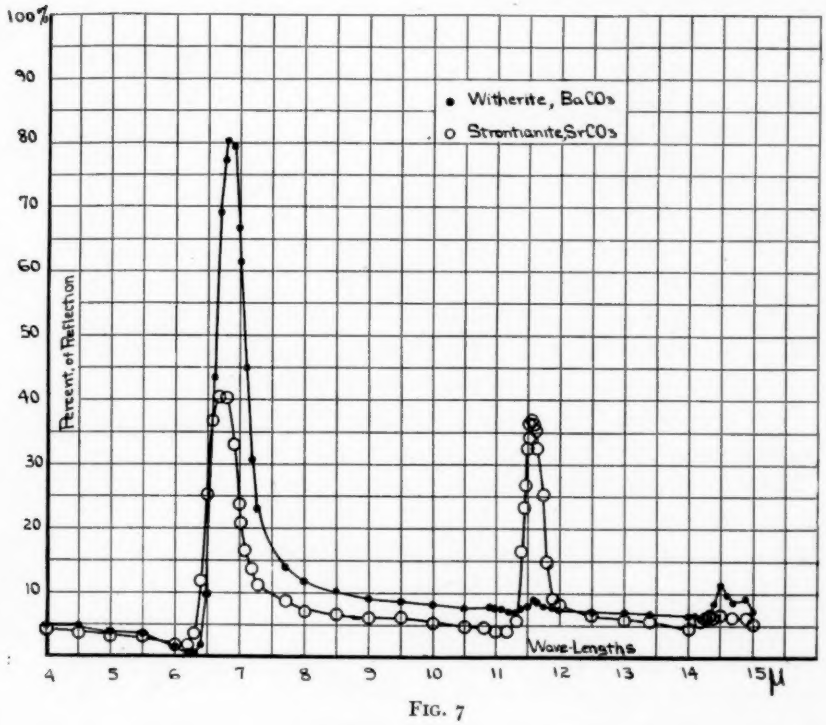


FIG. 7

proportional to the change in the atomic weight of the base. The smithsonite ($ZnCO_3$) band in the third region shows the greatest deviation.

A second specimen of smithsonite gave a band shaped differently, especially in the second region, but having practically the same maxima as the first in all three regions. Its band in the third region was several times as broad as the calcite band and had its maximum on the side toward the short waves. With the exception of the magnesite,

the other points showing the greatest deviation from the straight lines have been mentioned in the third preceding paragraph. These lines drawn through the cerussite and calcite (*Pb* and *Ca*) maxima, to aid in comparing the results, are practically parallel. All but three of the eighteen points in the first two regions lie near these parallel lines, which indicates that the average displacement in each region due to the same change in atomic weight of the base is of the same order of magnitude.

Half of the points in the third region are considerably off the line, but, as will be shown later, a larger allowance for errors must be made in this part of the spectrum. The reflection curves usually published where a rock-salt prism is used end before the beginning of these bands because of the rapid increase, with wave-length, in the absorption of rock salt in this part of the spectrum.

In the comparison of maxima it is well to recall that a band's position may be influenced by impurities in the specimen, by the selective emission of the source, or by the selective absorption of any medium in the path of the beam. A small per cent. of a salt with a strong reflection band near the true band of the carbonate might easily serve to broaden the band and shift its maximum, especially when the true band is of low intensity.

Errors.—In some respects the position of two of the three regions is rather unfortunate. In the first region the intensity of the emission of a Nernst glower varies[1] both rapidly and irregularly with change in wave-length, and water vapor has an absorption[2] band with a maximum at $6.1\ \mu$. In the third region the rapidly increasing absorption of the rock salt used as a prism and radiometer window would tend to displace the apparent position of maxima toward the short wave-lengths by an amount depending upon the form of the band. With slits 1 mm in width the reflection from silver at $14\ \mu$ was 119 divisions and but 47 divisions at $15\ \mu$. Some of the bands in this region were so low that it was possible to detect them only by repeating observations at short wave-length intervals, and when found it was difficult to determine the precise position of the maximum. The uncertainties besetting measurements here are further

[1] W. J. H. Moll, *Onderzoek van Ultra-roode Spectra*, Plate VIII, Utrecht, 1907.

[2] The total path of the beam in air was 2.7 meters.

increased by the wide slits[1] employed. The dispersion theory calls for a sudden drop just preceding a rise in reflection and, in nearly all cases, the curves show this in the third, as well as the two earlier bands.

Complexity of bands.—Irregularities in the shape of nearly all the bands in the first region, which were prominent when the points observed could all be plotted on a larger scale, are still quite distinct in the curves shown, especially in siderite and rhodochrosite. These irregularities, together with Coblentz's observation of two maxima in the first bands of calcite and magnesite suggest that a higher resolving power would show all the carbonate bands in the first region to be complex.

Few irregularities in the shape of bands were observed in the second region, but several appear in the third, though here the deflections were so small that it was difficult to distinguish between small errors in observation and true irregularities in the curve. The impurity of the spectrum resulting from the wide slits necessary in the study of the second and third bands may have not only depressed the maxima but concealed the details of any characteristic structure within the bands.

Structure, etc.—In general the data on record do not indicate that differences in the position of bands in substances with the same chemical composition should be expected so far out in the infra-red. But, if both aragonite and calcite, both calcium carbonate, are reasonably pure, as their structure would indicate, such a difference exists in the reflection from their crystals. The second bands differ considerably both in their position and in their magnitude. Moreover, the higher reflection of aragonite in the second region cannot be attributed to a surface difference, as calcite shows the higher reflection in the first.

No classification of the results according to the chemical group to which the base belongs has been possible. If displacements due to this are present they must either be irregular or secondary in magnitude to those produced by a change in the atomic weight. Neither has any simple relation been found to exist between the wave-lengths

[1] In the first, second, and third regions, slits 0.3 mm, 0.8 mm, and 0.1 mm in width were generally used.

of the regions within which the bands fall. For convenience of comparison the wave-lengths of the reflection maxima are given in Table I.

TABLE I

Substance	Chemical Composition	Atomic Weight of Base	Reflection Maxima		
			Band 1	Band 2	Band 3
Magnesite	$MgCO_3$	24.2	6.5 μ	11.2 μ	13.9 μ
Calcite	$CaCO_3$	39.7	6.6	11.31	14.2
Aragonite	$CaCO_3$	39.7	6.65	11.55	14.2
Rhodochrosite	$MnCO_3$	54.6	6.63	11.47+	14.0−
Siderite	$FeCO_3$	55.5	6.60	11.47−	13.9−
Smithsonite	$ZnCO_3$	64.9	6.7	11.38	13.6
Strontianite	$SrCO_3$	86.9	6.76	11.56	14.37
Witherite	$BaCO_3$	136.4	6.86	11.60	14.5
Cerussite	$PbCO_3$	205.4	7.2	11.94	14.8

EARLIER DATA IN AGREEMENT

The earlier data on the reflection of the anhydrous salts of carbonic and other simple acids, though meager, are still in agreement with the conclusions drawn from carbonates concerning the shift of bands with change in the atomic weight of the base.

Carbonate.—In the data of Coblentz on calcite and magnesite the wave-lengths of both components of the complex band in the two substances lie in the order of the atomic weights of the bases. Although Coblentz gives the reflection of magnesite to 12.4 μ, his observations were at such long wave-length intervals that he missed the second reflection band which was doubtless present because the specimen used, judging from the height of the first band, was superior to the one here described.

Nitrate.—Pfund found that KNO_3 $(K=38.9)$ and $AgNO_3$ $(Ag=107.1)$ had bands at 7.05 μ and 7.45 μ respectively, shown by crosses in Fig. 8. Coblentz's value for KNO_3, 7.15 μ, is shown by a circle.

TABLE II

Substance	Chemical Formula	Atomic Weight of Base	Maxima for First Band		
Anhydrite	$CaSO_4$	39.7	....	8.6 μ	9.1 μ
Celestite	$SrSO_4$	86.9	8.2 μ	8.76	9.1
Barite	$BaSO_4$	136.4	8.35	8.9	9.1

Sulphate.—The maxima in the reflection of simple anhydrous sulphates are given in Table II, compiled from the data of Coblentz.

In each case the maximum in the middle column is the highest and corresponds more closely to the center of the complex band. These values are plotted with the atomic weight of the base in Figure 8,

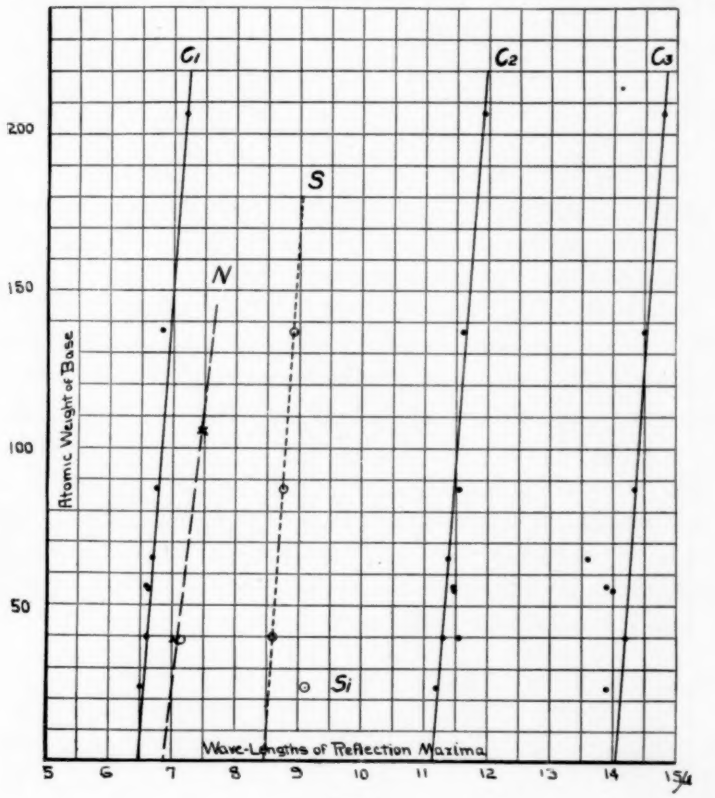


FIG. 8

and the line *S* drawn connecting these points is practically straight, showing that the displacements are proportional to the change in the atomic weight of the base, which agrees with the statement made regarding the displacement in carbonates. The line *N*, through the nitrate points, and the line *S*, through the sulphate points, are both approximately parallel to the lines, C_1, C_2, and C_3, drawn for the three carbonate bands. From this we are led to suspect that the rate of

shift of the band with increase in the atomic weight of the base is of the same order of magnitude in carbonates, nitrates, and sulphates, though more complete data will be necessary to determine the exact relations and perhaps the significance of the different oxygen content of the acid radicals.

Silicate.—The circle[1] marked Si represents the position found for a band in enstatite ($MgSiO_3$).

II. THE RÔLE PLAYED BY OXYGEN IN THE SELECTIVE REFLECTION CHARACTERISTIC OF CARBONATES, NITRATES, SULPHATES, AND SILICATES

In this connection a somewhat broader phase of the subject presents itself, and we ask, Do the bands in these different acid radicals have any relation to each other?

The selective reflection of the salts (given in Table III) with bases having nearly the same molecular weight is compared. In Fig. 9 the weights of the elements combined in the acid radical with three molecules of oxygen are plotted as ordinates and as abcissae the wave-lengths of the first reflection bands. A straight line fits the results remarkably well, especially when one recalls that the lower atomic weight of magnesium is partly responsible for the highest point lying toward the short waves, and when one takes into account the difference in the values obtained by independent investigators for the KNO_3 band shown in the same figure (Fig. 9).

TABLE III

Substance	Chemical Composition	Atomic Weight of Base	Weight with 48 g of O	Position of Band	
Calcite................	$CaCO_3$	39.7	12g of C	6.6 μ	
Potassium Nitrate......	KNO_3	38.9	14 N	7.15*	7.05†
Anhydrite.............	$CaSO_4$	39.7	24 S	8.6[1]	
Enstatite..............	$MgSiO_3$	24.2	28 Si	9.1[1]	

* Coblentz, *loc. cit.* † Pfund, *loc. cit.*

A change in the weight of the element combined with equal amounts of oxygen in the acid radical produces a much larger shift in the position of the reflection maximum than is produced by the same change

[1] Coblentz's data in Fig. 8 shown by circles, Pfund's by crosses.

in the atomic weight of the base. This is in agreement with the chemist's views regarding the relative strength of the bond existing in the two positions. Similar considerations suggest that peculiarities of the individual elements other than the differences in their atomic weights may be found to exert a stronger control when within the acid radical

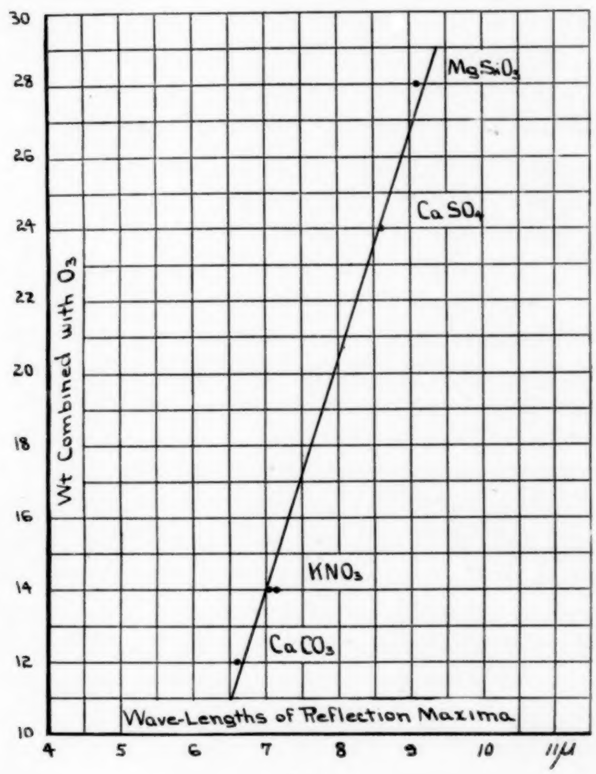


Fig. 9

than they do in the base, and similarity or dissimilarity of its relation to the oxygen in the acid radical must be carefully studied.

In this we may have a clue to a general method by means of which chemical formulae may come to have a more definite and wider dynamical meaning than they now do. At present, atomic relations within the molecules of a solid can only be inferred from evidence gained primarily from solutions and gases.

From the foregoing results it appears that there is a regular shift in the reflection bands, characteristic of a given acid radical, with a change in the atomic weight of the base with which it is combined; and second, that the active and characteristic element in the carbonic, sulphuric, nitric, and silicic acid radicals is oxygen. A change in the atomic weight of the element, combined *directly* with the *oxygen* in the acid radical, is far more potent in shifting the wave-length of the reflection band, than the same change in the atomic weight of the base, which is *indirectly* combined with the oxygen.

To establish the second hypothesis on as firm a footing as the first now rests, it may be necessary to give to salts of other acids the same systematic study which the carbonates have received, and in doing this the writer is at present engaged.

SUMMARY

1. The reflection curves for all the carbonates examined show between 4 μ and 15 μ three, and only three, bands of marked reflection.

2. The bands fall into three separate and definite spectral regions, which are distinct from the regions where the salts of other acids, so far as known, show reflection maxima.

3. With few exceptions, an increase in the atomic weight of the base causes a shift of all three reflection maxima toward long waves by an amount roughly proportional to the change in atomic weight of the base.

4. No regular displacements traceable to the chemical group to which the base belonged were observed, nor does any simple relation appear between the wave-lengths of the three bands in carbonates.

5. Combining with the data on carbonates the scattered observations on nitrates, sulphates, and silicates, the tentative hypothesis has been made that the oxygen atom is the one chiefly responsible for the marked reflection observed.

6. The results presented in the present paper suggest a new and far-reaching method by which it may some time be possible to express the dynamical relations existing between the separate atoms of a molecule, and thus the present conception of chemical bonds and linkages be given a broader significance.

In conclusion, the author wishes to thank Professor E. F. Nichols,

who suggested the problem and directed the work, for the daily interest shown; but he would like especially to express his deep appreciation of the profit he has derived from frequent discussions during the progress of the work concerning its broader relations.

PHOENIX PHYSICAL LABORATORY
Columbia University
August 1907

ADDENDUM

Since the foregoing paper went to the printer my attention has been called to a short paper in the *Jahrbuch der Radioaktivität und Elektronik* (**4**, 132, 1907) by Dr. W. W. Coblentz in which, after reporting the analysis of the first reflection band in eight carbonates and the reflection and absorption bands between 4μ and 9.4μ in seven sulphates, diagrams are given and the following conclusions drawn:

> Im ganzen genommen reichen die dargestellten Ergebnisse hin, um nachzuweisen, dass das Molekulargewicht tatsächlich die Lage des Maximums beeinflusst; es ist aber zu beachten, dass die Verschiebung durch das metallische Atom oder "Ion" verursacht wird an welche die Atomgruppe gebunden ist. Andererseits wird, nach früheren Ergebnissen, die Lage des Maximums durch die Anzahl der Atomgruppen nicht beeinflusst. Dasselbe gilt auch für das Kohlenstoffatom an sich. Die Atomgruppen sind mithin als die Ursache gewisser charakteristischer Absorptions- und Reflektionsbanden zu betrachten; die Lage dieser Banden aber wird durch das Atomgewicht des metallischen Atoms bestimmt, mit dem vereinigt die Atomgruppe die Verbindung bildet.

Dr. Coblentz's paper is dated March 22, 1907. Before this, a considerable portion of the data here presented had been gathered.

L. B. M.

AN ABSOLUTE SCALE OF PHOTOGRAPHIC MAGNITUDES OF STARS

By J. A. PARKHURST and F. C. JORDAN

The determination of star-magnitudes by the method of measurement of the opacity of the silver deposit on extra-focal images, makes it possible to obtain an "absolute" scale; that is, the effect on the plate of lights differing by a known ratio can be determined by laboratory experiments. If the results obtained by this method were no more accurate than those given by indirect methods (such as the use of a number of stars of known magnitude on each plate) it would still be valuable as a check on those results. As a matter of fact, the opacity-measure of extra-focal images yields results of somewhat greater precision than is usually obtained by other methods. Added importance thus attaches to the correct determination of an "absolute" scale.

The method used in Europe of impressing on each plate, besides the ordinary image of a star, one formed through a wire grating or "Gitter," has its disadvantages, some of which are avoided by the method about to be described, which is very simple in theory and only requires the observance of suitable precautions in order to yield results of considerable accuracy.

The procedure is to illuminate certain areas of a plate simultaneously by lights differing in intensity by a known ratio. In this way the time element, or the truth of the so-called "law of reciprocity," does not enter the problem; and, as we are not dealing with the diameter of star images, the disturbing effect of any change of the source of illumination, or in the path of the rays, need not be considered.

To obtain the desired illumination a sensitometer box was used, shown in Fig. 1, eight inches long, holding a 4x5 plate at each end. Running lengthwise of the box are 42 light-tight cells, seven inches (18 cm) long and one-half inch (13 mm) square. One end of this system of cells was covered by a metal plate pierced with a hole opposite the center of each cell. When this end of the box was uniformly illuminated, the amount of light passing through each cell was fixed by

PLATE XI

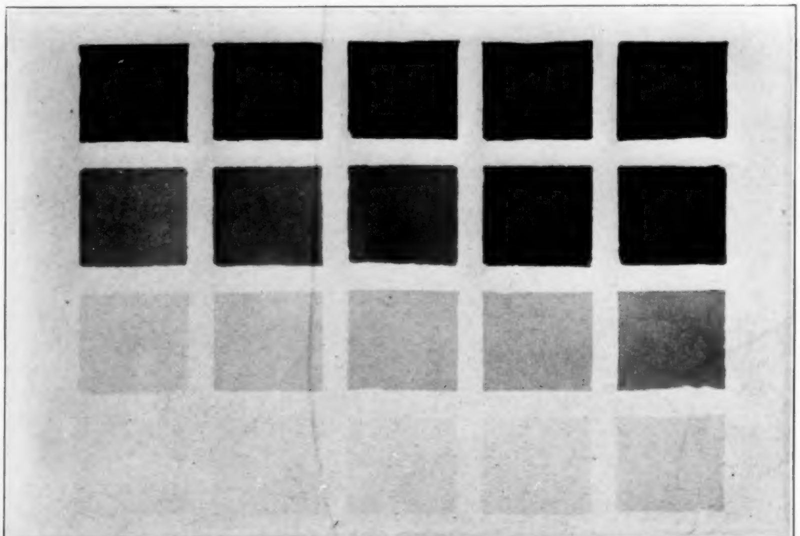


FIG. 2

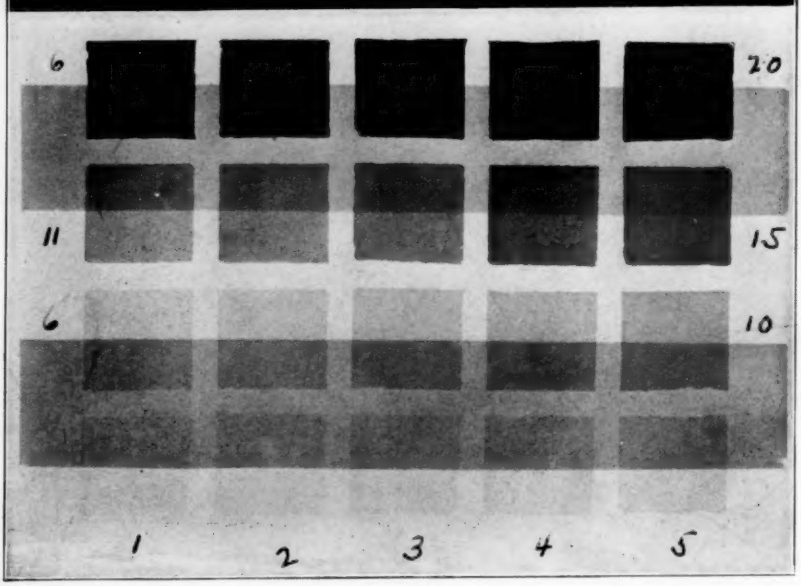
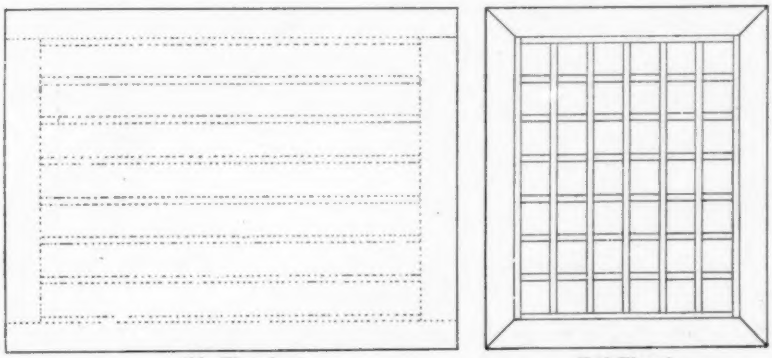


FIG. 3

SENSITOMETER PLATES

the diameter of the hole. To insure uniformity of illumination, two or more pieces of ground-glass, one inch apart, were put between the source of light and the box. A sensitive plate placed at the other end of the box will be blackened in squares corresponding to each cell, the opacity of the deposit depending on the amount of light admitted by the hole in the metal plate. Fig. 2 (Plate XI) shows a specimen



Side Elevation | End Elevation

FIG. 1.—Sensitometer Box

plate, No. 47 in the series taken with metal plate *D*, on which only the 20 inner cells were used. Table I gives the numerical data for metal plate *D*.

TABLE I

No.	Diam.	Relative Area	Δ log Area	Δ Mag.	No.	Diam.	Relative Area	Δ log Area	Δ Mag.
1....	1.035	1.071	0.000	0.000	11....	2.639	6.963	0.813	2.033
2....	1.107	1.225	0.058	0.146	12....	2.827	7.992	0.873	2.182
3....	1.212	1.469	0.137	0.343	13....	3.076	9.462	0.946	2.366
4....	1.330	1.769	0.218	0.545	14....	3.599	12.953	1.083	2.707
5....	1.535	2.356	0.342	0.856	15....	3.757	14.115	1.120	2.800
6....	1.633	2.667	0.396	0.991	16....	4.186	17.523	1.214	3.035
7....	1.841	3.389	0.500	1.251	17....	4.647	21.586	1.304	3.261
8....	1.982	3.928	0.564	1.411	18....	5.002	25.020	1.368	3.421
9....	2.116	4.477	0.621	1.553	19....	5.407	29.236	1.436	3.590
10....	2.391	5.717	0.727	1.819	20....	6.271	39.326	1.565	3.912

In this table the diameters are expressed in millimeters, the "relative area" is the square of the diameter, the "Δ log Area" is the difference between the log area of each hole and that of hole No. 1, and finally the "Δ Mag." is the Δ log area divided by 0.4. This last column

will therefore represent the relative star-magnitudes of the lights passing through each cell.

For the measurement of these opacities and the star plates on which the method has been applied, a Hartmann "mikrophotometer"

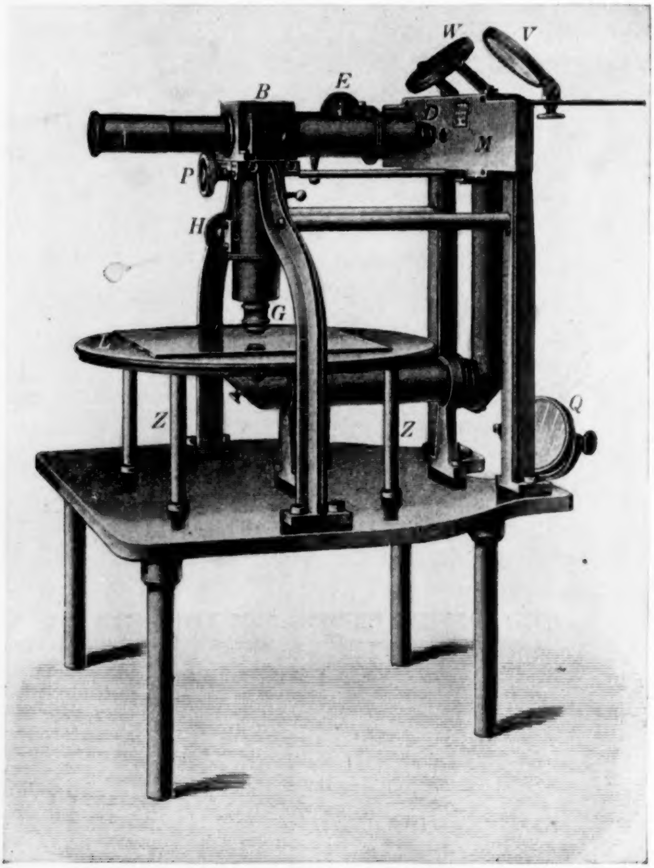


FIG. 4

has been used. The purchase of this fine instrument was made possible by a grant from the Rumford Committee of the American Academy, and acknowledgment is here made of the aid thus kindly furnished for this work. A perspective view of the instrument is shown in Fig. 4 and a full description will be found in this journal,

10, 321, 1899. As there described, the method of measurement is to match the opacity with a photographic wedge. The wedge so far used is a portion of a plate furnished by Professor E. C. Pickering and numbered by him "E 5862." This is evidently one of the plates made by Mr. E. S. King[1] and similar to that used by J. A. Parkhurst in the equalizing wedge-photometer.[2]

It will now be evident that if a plate such as Fig. 2 be measured in the photometer, the absorption-curve of the wedge will be given directly in stellar magnitudes. Then if star images are taken out of focus, the effect of stars of different magnitudes in illuminating the image surfaces will be comparable with the light passing through holes of different diameters. Therefore the scale of the photographs will be "absolute" in the sense that it is derived from laboratory experiments.

Among the precautions used to insure consistent results the following may be mentioned:

1. The same brand of plates, Seed 27, were developed with hydroquinone developer of the same constitution for ten minutes at $+20°$ C.

2. A test was made of the effect of exposure temperatures between $-2°$ and $+17°$ C. It was found that the development factor (the maximum slope of the absorption-curve) was not affected between these limits. Changes, if any, were confined to the thinnest and densest squares, and these were not used in the measures.

3. A test of different colors of light gave a like negative result. Incandescent lights burning above and below candle-power, daylight, and magnesium light gave the same absorption-curve. It should be noted, however, that for ordinary exposures the sensitiveness of the Seed plate extends only to about the $H\beta$ line, so that this test merely showed the effect of light of shorter wave-length than $H\beta$.

4. Changes in the exposure time between ten seconds and thirty minutes had no effect on the curve. Had this test given a different result the method could not be used on the stars, since the exposure times must vary according to the faintness of the stars required.

5. Negatives made on plate glass gave much more accordant results than those on ordinary glass. Local errors, due to the thickness of

[1] *Annals of Harvard College Observatory*, **41**, 237.

[2] *Astrophysical Journal*, **13**, 249, 1901; *Researches in Stellar Photometry*, 8.

the emulsion, were quite noticeable in the ordinary glass, but almost disappeared on plate glass.

6. Four metal plates, *A*, *B*, *C*, and *D*, each with a different arrangement of holes, were used with various exposure times, and the box was frequently inverted, so that the light from a particular hole, or cell, would fall at different parts of the absorption-curve. There were no systematic differences found from the various metal plates or the different positions of the box. Fig. 5 shows the final absorption-curve used in the reductions. It is derived from the measures of twenty plates, four of them being on plate glass, all taken with metal plate *D*.

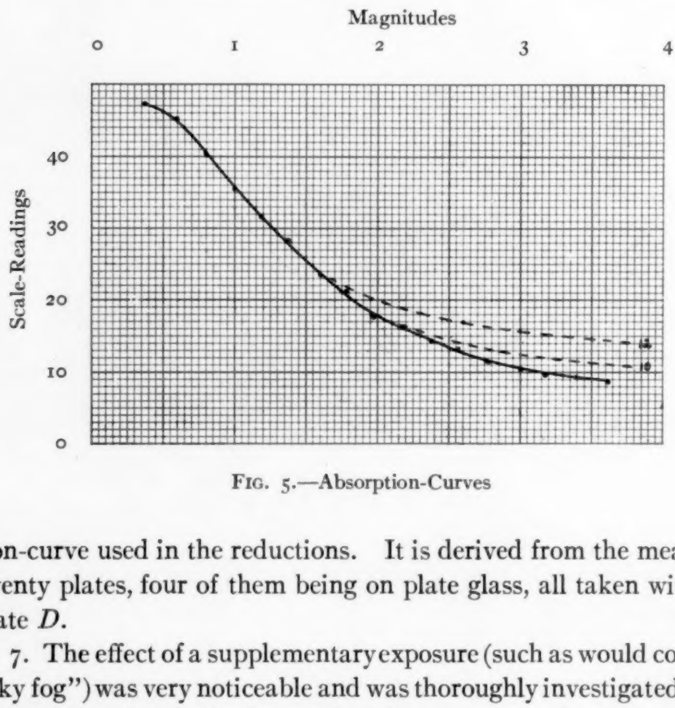


FIG. 5.—Absorption-Curves

7. The effect of a supplementary exposure (such as would come from "sky fog") was very noticeable and was thoroughly investigated. Strips five-eighths of an inch (15 mm) wide were exposed across the plate, fogging half of each square, so that the clear and the fogged part could be measured. This arrangement is shown in Fig. 3. Plates were fogged before, during, and after the main exposure, giving the same result. The effect of a very slight fog was noticeable on the thin squares, but it was not measurable on the denser squares, even when the fogging was as great as would result from three hours' exposure in the camera to a dark sky. It is therefore evident that the star plates cannot all

be reduced with the same absorption-curve, but the amount of fogging in the film can be readily measured with the photometer, and in practice each plate is reduced with a curve corresponding to the fog. The broken lines in Fig. 5 show the curves corresponding to fog readings of ten and twelve on the photometer scale.

The stellar plates on which this method is applied, have been taken with a Zeiss doublet of 14.5 cm aperture and 81.4 cm focal length. Most of the plates have been taken 7 mm inside the focus, giving a star image 1 mm in diameter. At this setting the illumination of the image is very nearly uniform, so that it can be measured in the photometer as accurately as a perfectly uniform area. With a disk of this diameter ten minutes' exposure gives a measurable image of a seventh magnitude white star, though a sixth magnitude star can be measured with greater accuracy, since the opacity falls at a steeper part of the absorption-curve. As tests and illustrations of the method, results of the measures of the *Pleiades* and a number of variable stars of short-period and *Algol* type will be given.

Pleiades

Of the countless measures of the *Pleiades*, the best for comparison with the present method seem to be those made by Schwarzschild,[1] both from the excellent quality of the work and the fact that they were measures of extra-focal photographic images, reduced by means of the visual magnitudes of the white stars. Four exposures on the *Pleiades*, of 1, 3, 10, and 25 minutes respectively, containing 19 stars, were measured and the absorption-curve of the wedge was platted, using Schwarzschild's magnitudes given in Table 14 of the work cited. Table II gives the differences between this curve and that derived from the standard sensitometer squares, expressed in magnitudes, for each 5 mm of the scale, in the sense, Sch.–P.

TABLE II

Scale	Δ Mag.	Scale	Δ Mag.
45	−0.20	25	0.00
40	−0.08	20	+0.01
35	−0.01	15	+0.01
30	0.00	10	+0.13

[1] *Publicationen der v. Kuffnerschen Sternwarte*, **5**, C.

The curves are almost identical between scale-readings 12 and 37, but there are systematic differences at both ends of the scale. These are probably due to the fact that readings on very thin images, below 12, are not reliable; further, the normal point at 42 depends on only two stars, and above that point the images are too dense for good measurement. The portion of the scale which can be used thus lies between 12 and 40, corresponding to 2.0 magnitudes. A very slight difference in opacity is readily recognized in the photometer, so that the average deviation of a single setting from the mean of three is between 0.1 and 0.2 mm, corresponding to a little more than 0.01 magnitude. From the above discussion it seems evident that within the limits mentioned the scale is correct, and the method is capable of yielding results of extreme accuracy over a range of about two magnitudes on a single plate. Certain useful lines of work are indicated, such as measurement of the light-curves of variables of the *Algol*-type and of short-period variables, especially those regarding whose changes there is conflicting evidence.

MINIMUM OF THE *Algol*-TYPE VARIABLE *U Ophiuchi*

Five plates were taken of the star, covering two minima. Fig. 6 shows the minimum platted from sixteen exposures on Plate 195 and

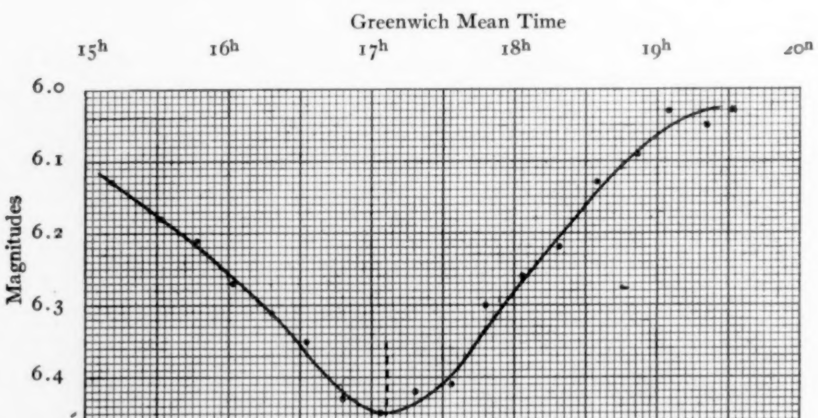


FIG. 6.—Minimum of *U Ophiuchi*, 1907 June, 13

two on Plate 196. The largest residual from the curve is 0.03, the average ± 0.014 magnitude. The total exposure on Plate 195 was 152 minutes, causing a sky-fog a little too dense for the reduction-curves; therefore the range shown is too small by 0.10 or 0.15 magnitude. The shape of the curve and the time of minimum are, of course, unchanged. The time of minimum is ~~$11^h 6^m$~~ Central Standard Time, or $17^h 6^m$ G. M. T. Reduced to the sun this becomes $17^h 14^m$. The correction to Hartwig's ephemeris in the *Vierteljahrsschrift* is $-1^h 0^m$. The correction to the ephemeris in the *Annuaire* is $+0^h 20^m$.

LIGHT-CURVE AND ELEMENTS OF *RZ Cassiopeiae*

This *Algol*-type variable was discovered by Müller and Kempf[1] who gave the elements of minimum 1906, May 24, $10^h 15^m + 1^d 4^h 40^m.8$ E. Its binary character is shown by spectrograms taken by Hartmann[2] at Potsdam and by Parkhurst with the Bruce spectrograph at this observatory.[3] Since the announcement of variability, eighteen plates of the region have been taken, containing fifty-eight exposures; covering besides the minimum the entire period. The comparison stars used are

B. D.	Potsdam Color	Potsdam Mag.	Spectrum	Adopted Mag.
F $+67^\circ.224$	GW	6.15	A	6.15
D $+69.171$	—	—	A	7.43

Special care has been taken to select white stars for standards in order that the visual magnitudes may be taken without correction. With this in view plates of the region have been taken with a Zeiss objective prism of 15° angle, used in connection with the same camera with which the extra-focal plates are taken. The scale is sufficient to show clearly the type of spectrum, and furnishes a more accurate indication of the star-colors than can be found in the *Potsdam Photometric Durchmusterung* or the *Draper Catalogue*. The strength of the K line in these spectra is an excellent criterion of the color. In white stars it is absent or very faint, while in spectra called *F* in the

[1] *Astronomische Nachrichten*, **171**, 357, 1906.

[2] *Astronomische Nachrichten*, **173**, 101, 1906.

[3] *Astrophysical Journal*, **25**, 59, 1907.

Draper Catalogue classification the K and H lines are about equal. These *F* stars have a photographic magnitude about $\frac{1}{2}^m$ fainter than the visual. The relation between color and spectral type is being investigated in this connection, and provisional results will soon be published in this journal.

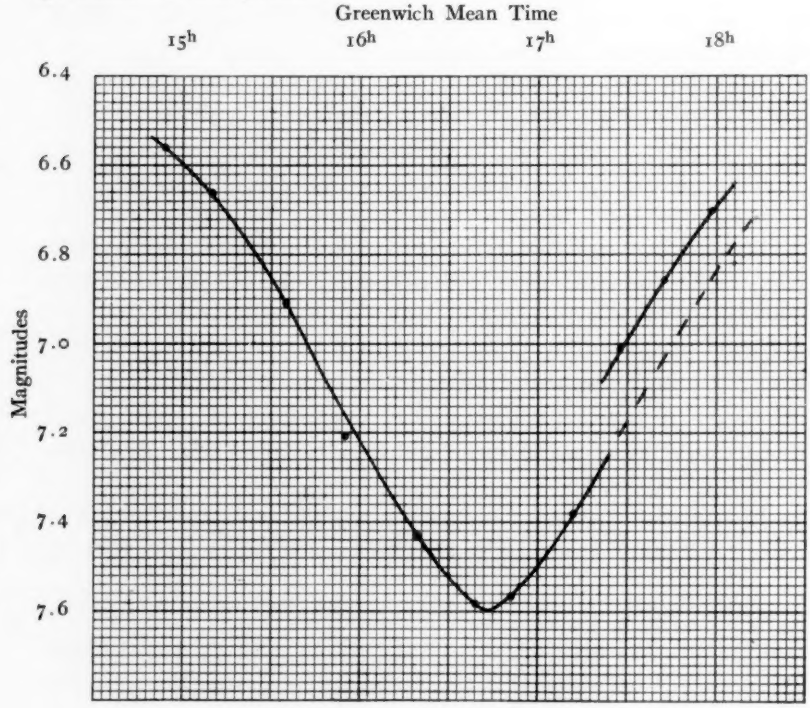


FIG. 7.—Minimum of *R Z Cassiopeiae*, 1907, Aug. 3

Fig. 7 shows the curve of minimum of 1907, Aug. 3, from Plate 210, having eleven exposures, the Greenwich Mean Times being 1907, Aug. 3 from $14^h\ 54^m$ to $17^h\ 58^m$, showing a minimum at $16^h\ 44^m$, which corrected to the sun is $16^h\ 42^m$. It will be noticed that the last three exposures give points on a curve about 0.2 magnitude above the rest. This is due to the overlapping of these images with those of the star *B. D.* $+69^\circ180$, which lies 52′ north of the variable. The probable curve which the star would have followed, were it not for this overlapping, is shown in the broken line. Fig. 8 shows[1] a part of

[1] The cut is not a good representation of the plate.

Plate 210, including the comparison stars *F* and *D*, also the star marked *E*, which is *B. D.* $+68^{\circ}200 = 155$. 1906 *Cassiopeiae*. The plate-holder is carried by a slide moved by a screw with a large head. The plate was moved one turn of this screw, $\frac{1}{20}$ inch, between the exposures, except

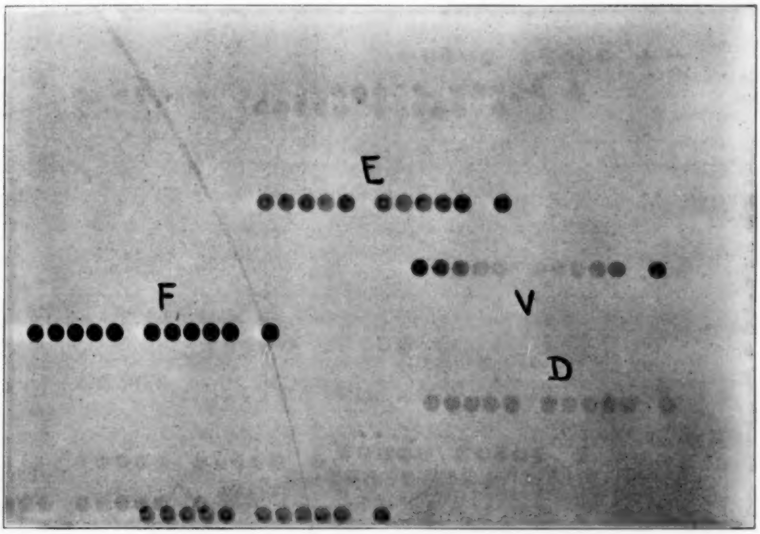


FIG. 8.—Extra-focal Images Showing Minimum of *R Z Cassiopeiae*

that two turns were used, after the fifth and tenth exposures, for aid in identification.

The entire light-curve of this star has been covered by eighteen extra-focal plates with fifty-eight exposures. Six of these plates occur in the minimum phase, and they have been combined with four focal plates taken by Jordan with the twenty-four inch reflector, to form a mean curve. Together they give the following normal points:

Time from Min.	Mag.	Time from Min.	Mag.
$-0^{d}106$	6.47	$+0^{d}017$	7.41
−0.072	6.64	+0.026	7.29
−0.054	6.84	+0.035	7.04
−0.030	7.25	+0.043	6.93
−0.009	7.55	+0.070	6.62
+0.002	7.64	+0.095	6.47
+0.014	7.61		

A smooth curve through these points gives the following mean curve

T.	Mag. Before	Mag. After	T.	Mag. Before	Mag. After
$0^d.00$	7.64		$0^d.06$	6.77	6.72
0.01	7.55	7.56	0.07	6.66	6.62
0.02	7.40	7.36	0.08	6.58	6.55
0.03	7.22	7.17	0.09	6.52	6.49
0.04	7.05	6.99	0.10	6.48	6.46
0.05	6.90	6.84	0.11	6.44	6.44

Fourteen plates with twenty-eight exposures give the following points in the normal light of the star:

Time after Min.	Mag.	Δ Mag. from 6.43	Time after Min.	Mag.	Δ Mag. from 6.43
$0^d.111$	6.45	+0.02	$0^d.673$	6.45	+0.02
0.174	6.47	+0.04	0.794	6.46	+0.03
0.299	6.40	−0.03	0.893	6.38	−0.05
0.520	6.44	+0.01	1.044	6.43	0.00

We may draw these conclusions from the above results:

1. The normal light is 6.43, the minimum 7.64, range 1.21 magnitude.
2. There is no trace of a secondary minimum.
3. The duration of the eclipse is 0.23 day, $=5^h\ 32^m$.
4. The minimum is sharply defined.
5. The maximum rate of change is 0.73 magnitude per hour.

The period seems to be about 22^s longer than that given by Müller and Kempf in the place above cited. The plates taken here are best satisfied by the elements:

Minimum = J. D. 2417355.427 + $1^d.195258$ E.

Following is a comparison of the residuals from our plates with the two values of the period.

Plate	Minimum Observed	Minimum Red. to Sun	Minimum Corrected Min.	Julian Day	Residuals, O.−C. M. and K. P = 1.1950	Residuals, O.−C. P. and J. P = 1.195258
	1906					
R 69.....	Aug. $14^d\ 21^h.5$	$-1^m.2$	$14^d\ 21^h.5$	7437.896	+0.014	+0.002
R 83.....	Aug. 31 $15^h\ 7^m$	+0.2	31 15 7^m	7454.630	+0.018	−0.003
R 104....	Sept. 25 17 35	+2.7	25 17 38	7479.735	+0.028	+0.001
R 175....	Dec. 19 14 0	+4.8	19 14 5	7564.587	+0.035	−0.010
	1907					
UV 365...	Aug. 3 16 44	−2.2	3 16 42	7791.696	+0.094	0.000
R 406....	Sept. 21 16 42	+2.2	21 16 44	7840.697	+0.100	−0.004

THE SUSPECTED VARIABLE *32 Cassiopeiae*

This star received the provisional notation 186.1904, and later the notation *RU Cassiopeiae*, but most observers have found it constant. A previous report by the writers[1] from 72 exposures on four reflector plates stated that the star was found constant at 0.03 mag. fainter than the neighboring white star $+63^\circ$ 149. As the extra-focal method seems particularly well adapted to settle such disputed questions involving slight variation, Plate 223 was taken of this field on 1907, Sept. 13, having eight exposures covering four hours.

Four comparison stars were measured, as follows:

Star	*B. D.*	*PDM* Color	*PDM* Mag.	Adopted Mag.
C..........	$+63^\circ$ 149	*GW*–	5.81	5.83
D..........	$+63^\circ$ 147	....	....	7.45
E..........	$+63^\circ$ 176	*GW*	6.64	6.55
F..........	$+65^\circ$ 115	*GW*–	6.10	6.15

Measures of *32 Cassiopeiae* on this plate gave

G. M. T.	Mag.	Δ from 5.86
$14^h\ 41^m$	5.86	0.00
15 19	5.87	0.01
15 30	5.86	0.00
16 2	5.86	0.00
16 19	5.84	0.02
17 2	5.85	0.01
17 39	5.86	0.00
18 34	5.85	0.01
Means	5.86	±0.006

It seems therefore to be reasonably certain that at the present time the star is not varying.

YERKES OBSERVATORY
September 1907

[1] *Astrophysical Journal*, **23**, 88, 1906.

TEMPERATURE CONTROL FOR SILVERED SPECULA

By HEBER D. CURTIS

In recent years, with the growing use of silvered glass specula in astronomical research, the subject of focal changes in such systems due to changing temperature has assumed considerable importance.

Professor Keeler, to whom, more than to any other, is due the establishment of the great power of reflectors in astronomical photography, has described some difficulties of this nature in his well-known paper on the Crossley reflector.[1] He did not consider that the changes which he found in a long exposure were due to temperature effects, but rather to the fact that the axis of the mirror, through flexure effects in the mounting, wandered irregularly over the field. As the field has considerable curvature for this angular aperture (1:5.8) the effect of a change in the focus was produced. His explanation that the focal changes in the Crossley reflector were thus inherent in the method of mounting, rather than in the mirror itself, is borne out by the experience of Dr. C. D. Perrine, who finds that with the new and greatly improved mounting of this instrument,[2] no appreciable change of focus is experienced which can be attributed to temperature effects.

Quite recently Director Hale has described similar difficulties in the use of the Snow horizontal telescope at the Carnegie Solar Observatory on Mount Wilson.[3] In this case the mirror is twenty-four inches in diameter with much smaller angular aperture, 1:30. The focal changes in this instrument have at times amounted to as much as twelve inches, and on one occasion the difference in focus was three inches for opposite limbs of the sun. Considerable improvement has been brought about by the use of electric fans to keep the air about the mirror in circulation, by taking photographs quite early or quite late in the day, and by carefully screening the mirrors till just before

[1] *Astrophysical Journal*, **11**, 325, 1900.

[2] *Lick Observatory Bulletin*, **3**, 124, 1905.

[3] *Astrophysical Journal*, **23**, 6, 1906; *Contributions from the Solar Observatory*, No. 4.

the moment of exposure. In the Snow telescope the phenomenon is doubtless complicated by the fact that there are two additional plane mirrors in the coelostat train. That a due proportion of the effect may rest in the flats is borne out by the experience of Professor Barnard at the Sumatra eclipse. In using a camera of 61.5 feet focal length he found an occasional variation of about six inches which he attributed to a change in the figure of the coelostat flat due to the action of the sun's rays.

In the work of the D. O. Mills expedition to the Southern Hemisphere Professor W. H. Wright found similar small focal changes in the 37-inch Mills reflector, due to temperature effects. The forthcoming report of the results secured by the expedition during the first three years will contain Professor Wright's data on these points, together with a theoretical discussion of the causes. For the purposes of this paper it will be sufficient to repeat here the main facts with regard to the instrument, which is of the Cassegrainian form.

The great mirror is of very clear glass, free from noticeable bubbles or defects, and has a clear aperture of 36.56 inches (92.9 cm). Its focal length is 17.46 feet (5.49 cm). A hyperbolic secondary mirror gives to the instrument an equivalent focus of 55.4 feet (16.89 meters). The disk of the large mirror is 5.5 inches (14 cm) in thickness at the center and is pierced by a central hole 4.87 (12.4 cm) in diameter. The cell is of cast iron about half an inch thick, the bottom of which has an opening 8.5 inches (21.6 cm) in diameter, which was ordinarily kept closed with a cast-iron filler having a two-inch aperture. Aside from this opening in the back of the cell, and the few small holes for the adjusting screws of the mirror support, the only other ventilation about the mirror was provided by a door six inches square in the side of the cube above the mirror.

Professor Wright found a progressive focal change of fifteen to twenty-five millimeters occurring in the first half of the night. This change was always in the direction of increasing focal length, it being necessary to lengthen the focus of the telescope gradually throughout the first four or five hours of a night's work. These changes Mr. Wright attributes to a more concave form of the mirror brought about by the fall in temperature.

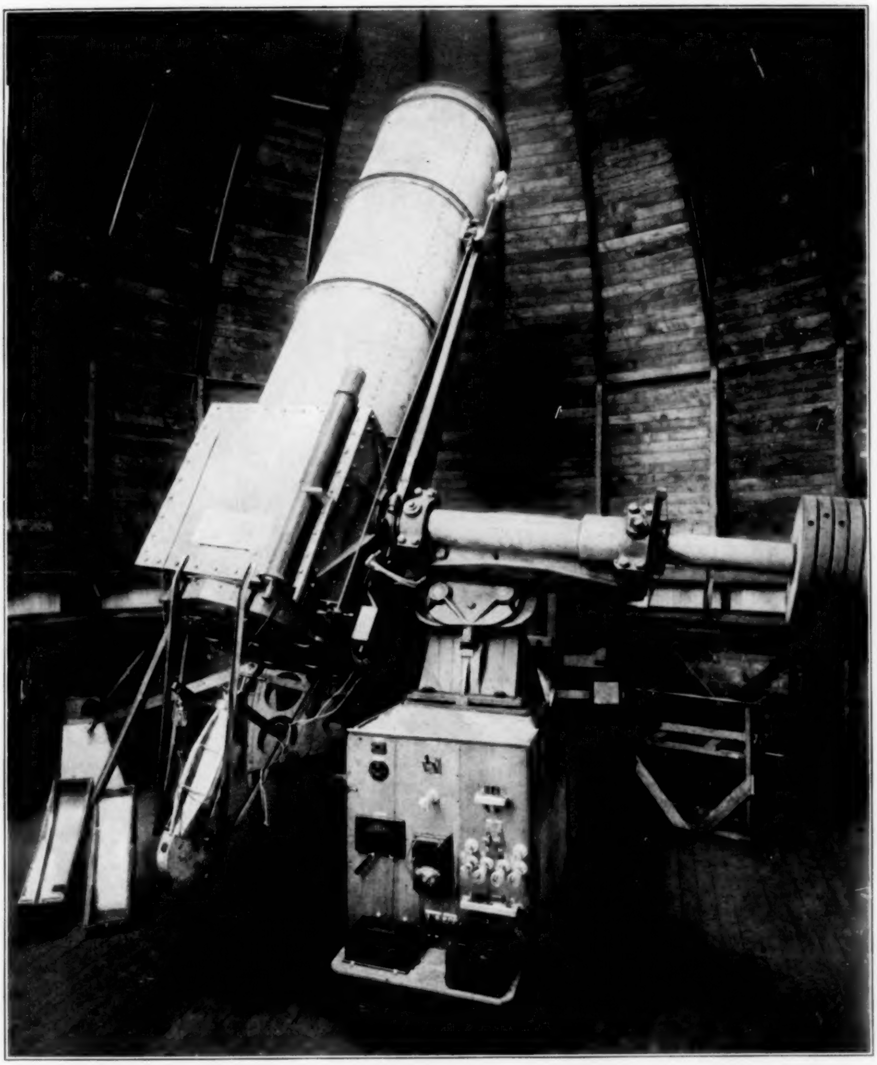
When the writer was appointed to continue for five years the work

in the Southern Hemisphere, for which Mr. Mills had generously made provision, it was decided, with the cordial support of Director Campbell, to try the effect of artificial cooling of the primary mirror in the effort to do away with these focal changes.

As a preliminary, the ventilation of the mirror in its cell was improved. The mirror cover, which had formerly rested nearly in contact with the glass, was moved fourteen inches up the cube; the small window in the cube was enlarged to six by sixteen inches and a similar window cut on the opposite side. Six holes, each 5.2 inches in diameter, were cut in the back of the cast-iron cell. The use of the iron filler for the central opening in the cell was discontinued. The area of the ventilating apertures at the back of the cell is thus about one-sixth of the area of the mirror.

During the past observing season a record has been kept of all the focal changes occurring in the mirror system. It cannot be said that the increased ventilation has in itself had much effect on the focal variations. These are perhaps a little smaller than before, averaging about twelve to fifteen millimeters under normal observing conditions. It is probable, also, that a condition of equilibrium is reached somewhat earlier, generally by four hours after sunset. Under normal summer observing conditions on Cerro San Cristobal a drop of 5° or 6° C. is experienced between three and eight P. M., followed by a slowly and generally very regularly decreasing temperature till dawn. In the less settled weather of fall and winter the daily range is generally smaller. The phenomena connected with the response of the mirror to the fall in outside temperature seem to be quite complex; many factors enter into the adjustment of the mirror to its condition of equilibrium, particularly the circulation of air in the dome and about the telescope. From a comparison of the observed focal ranges with the temperature records, it has not been possible to deduce any accurate relation between the two. Not infrequently a daily range of 2° C. will produce as great focal changes as a range of 6°. The rapidity with which a temperature change occurs seems to be of greater effect than the actual amount of the change. Neither has it been possible to establish that the focus of the system, when equilibrium has been reached, is different for different temperatures. With an irregular temperature-curve accompanying poor observing

PLATE XII



THE REFLECTING TELESCOPE AND THREE-PRISM SPECTROGRAPH OF THE D. O. MILLS EXPEDITION

conditions, the focal changes of the system are themselves quite erratic; in this class come the infrequent focal changes sometimes observed in the latter half of the night. As a rule, however, the focal changes between midnight and dawn are entirely absent.

Professor Wadsworth and Professor Ritchey have recommended the silvering of the back of mirrors to avoid such temperature effects. This plan has been tried with this mirror, but without appreciable effect on the focal range.

That the focal range has its origin entirely in the large mirror is proved not only by the results with artificial cooling to be given later, but also by focal tests of the primary on ten nights with an inclined photographic plate placed at its focus. On two of these nights with bad observing conditions and gusty wind the focal behavior was quite erratic; on the other nights a progressive focal range exactly parallel to that of the system was found, ranging from 1.2 to about 2.2 mm per night. The focal range of the Cassegrainian system is 10.1 times that of the primary, being the ratio of the squares of the two focal lengths. Using this multiplying factor we find a satisfactory agreement between the results of these tests and the focal ranges of the complete system. On two nights focal trails were made in quick succession at the focus of the primary, using alternately the central portion of the mirror and the outer zone. These tests showed that in the first part of the night the focus of the outer zone was shorter than that of the inner portion by about 1.2 mm, a difference which vanished in similar tests made at 3 A. M. Insulating the hole at the center of the primary with blanketing has not reduced the focal range.

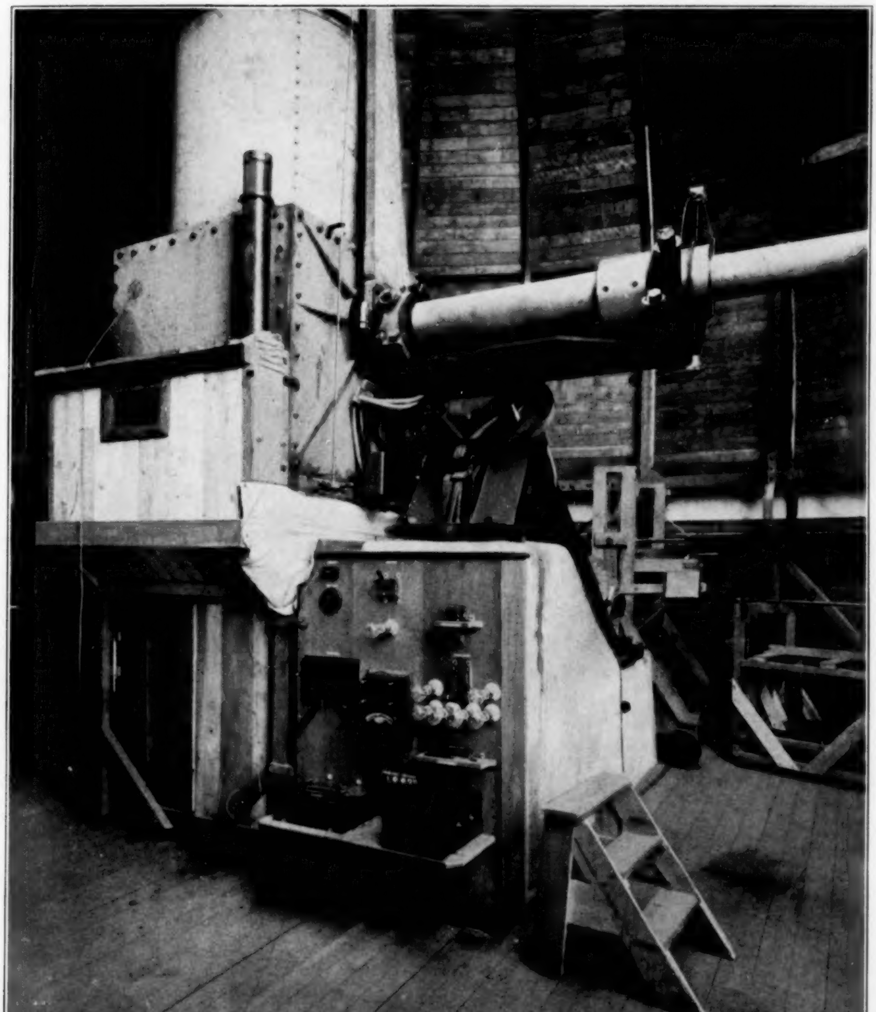
The method of artificial cooling adopted is that of refrigeration by anhydrous ammonia. The machine was made by the Brunswick Refrigerating Company, of Brunswick, N. J., and is the smallest size of their regular commercial line of self-contained refrigerating plants which they manufacture for isolated cooling equipments. It is what is designated on their scale as a one-hundred-pound machine, i. e., its capacity is approximately that equal to the melting of one hundred pounds of ice per day; and it can, in addition, make ten pounds of ice per day. The machine occupies approximately six by two and a half feet of floor space, and requires a one H. P. motor to run it and

the small pump used for circulating water through the condensing system.

The operation of the machine is, in brief, as follows: liquid anhydrous ammonia is allowed to expand from its reservoir into the cooling coils, an automatic expansion valve providing that the pressure in the cooling system shall not rise above twenty pounds to the square inch. From the cooling coils the gaseous ammonia is withdrawn by the ammonia pump, compressed to a pressure of from one hundred and fifty to two hundred pounds, depending upon the external temperature and the temperature of the water in the condenser; the water circulating through the condenser takes up the heat produced by the condensing of the gas, so that when it reaches its reservoir again, it is about at the temperature of the outside air liquefied, at least in part, and ready to pass again through its cycle of alternate expansion and compression. The machine is entirely automatic in its action, it being necessary only to turn on the water circulation, open two valves, and start the motor.

As installed on Cerro San Cristobal the refrigerating machine is located in the small workshop, at a distance of forty-eight feet from the telescope pier to one side of which the cooling coils are permanently attached, being insulated from the pier by a layer of wood and two of heavy felt. These coils are of one-inch iron pipe, occupy a space three feet by two feet by six inches, and are connected with the refrigerating machine by strong iron pipe of about one-quarter-inch bore, the pipe being carefully insulated with cork or felt insulation, and all joints carefully soldered. When the mirror is being cooled, the telescope is placed in a vertical position and a removable wooden case rolled into position which is so arranged that the fastening of a few clips completely insulates from the outside air the interior of the case, containing the spectrograph, the mirror, and the lower half of the cube. The case itself is insulated within with thick felt and contains approximately eighty-five cubic feet, making no deduction for the spectrograph, mirror, or iron-work of the telescope. Two electric fans blow the cold air from the coils up so as to circulate freely around the mirror through the holes in the cell and in the cube. A double glass window allows readings to be taken on a thermometer placed inside the cube with its bulb close to the edge of the mirror. The

PLATE XIII



REFRIGERATING CASE IN POSITION

drop in the mirror temperature is slow for the first half-hour, owing to the fact that all parts of the piping and case must be cooled; later the drop is more rapid and no difficulty is found in lowering the temperature at the mirror from 5° to 7° C. in a run of one and a half hours.

The procedure which has been found to give the best results is to start the refrigeration about two and a half or three hours before sunset. After the thermometer at the mirror shows a fall of 5° or 6° C. the machine is stopped and the case removed at about forty minutes before sunset; at this time the outside temperature is falling quite rapidly, and the temperature of the mirror, at least of its outer portions, is somewhat below that of the air. By half an hour after sunset, under usual conditions, the mirror has adjusted itself perfectly to its focus for the night. Frost gathers thickly on the cooling coils, but no evidence has been found of any moisture forming on the silver surface, even when the mirror is two or three degrees C. below the temperature in the dome. A thick shield of felt and blanketing protects the spectrograph and its prisms from getting too cold as a result of the direct radiation from the frost-covered pipes. The procedure of leaving the mirror to adjust itself to equilibrium for a short time before using seems to give better results than planning the cooling to end at the time of beginning work. The difficulty here is to stop just at the right time; unless this is done there are slight focal changes for an hour or two.

In its control of focal changes this method of artificial cooling has been found to be quite successful. Focal changes are as a rule entirely absent; when occurring they are quite small, being rarely more than five mm. Sudden changes in the night temperature still cause small focal changes. Perhaps an open-work construction of the cell and cube so as to give very free circulation about the mirror might reduce these changes still farther; fortunately the temperature-gradient under average observing conditions here is very regular. When the cooling is employed, tests of the focus at sunset almost invariably show it to be at the same point at which it was left at the end of work on the night before. Occasional mistakes are made in the amount of cooling required when clouds or unusual conditions have greatly reduced the daily temperature range. On such occasions, when the

mirror has been cooled to a temperature considerably too low, it is interesting to note that the progress of the focal changes is reversed, being in the opposite direction from that observed in the uncooled system.

There is no evidence that the artificial refrigeration affects the silver surface of the mirror injuriously.

THE D. O. MILLS EXPEDITION
Santiago, Chile, June 1907

ORBIT OF THE SPECTROSCOPIC BINARY θ *DRACONIS*

By HEBER D. CURTIS

The binary nature of this star ($\alpha = 16^h\ 0^m.1$; $\delta = +58°\ 50'$) was discovered by Director Campbell.[1] Its visual magnitude is given as 4.1; and the photographic, as 4.8. Its type is described as *F* and XIII*a* in the Harvard classifications and as II*a* by Potsdam. Its lines are of poor quality, rather diffuse, and not easy to measure. This is particularly the case when the star is somewhat underexposed; for this reason six plates were given only half-weight in the discussion, and one plate, that of August 8, 1899, was rejected. In the table are given the plates and the velocities upon which the determination of the elements was made. All the plates were measured by the writer at Mt. Hamilton; the last nine plates were taken with the remounted Mills Spectrograph, λ 4500 central; the others with the original Mills Spectrograph, λ 4340 central.

No.	Plate	Date, G. M. T.	Velocity	Weight
			km	
1	680 D	1898, Mar. 23.978	+15.5	
2	696 C	April 6.986	−30.2	
3	1215 C	1899, April 8.978	+12.0	
4	1220 D	10.851	−14.3	
5	1221 A	10.900	−12.8	
6	1237 B	May 1.843	−32.4	
7	1275 A	June 8.852	+ 9.0	
8	1291 B	18.812	+ 7.3	
9	1309 B	27.819	+12.0	
10	1317 A	July 4.810	−24.0	
11	1324 B	11.728	−27.0	
12	1351 A	25.710	+ 5.4	
12*a*	1375 A	Aug. 8.707	−16.8	Rejected
13	2486 B	1902, Aug. 11.729	−17.2	½
14	2515 B	20.788	− 7.8	
15	2540 C	Sept. 14.743	−23.8	
16	2720 A	1903, April 5.954	−32.1	
17	2723 A	6.936	+ 5.4	½
18	2734 D	9.020	−30.8	
19	2745 E	29.002	+15.8	
20	2752 B	30.017	−23.8	½

[1] *Astrophysical Journal*, **9**, 311, 1899.

No.	Plate	Date, G. M. T.	Velocity	Weight
			km.	
21	2760 D	1904, May 4.997	+14.8	
22	2784 A	11.967	−15.3	½
23	2889 A	Aug. 12.781	−31.2	
24	3177 C	1904, Feb. 29.010	+13.9	
25	3210 F	April 12.012	−14.3	
26	3222 E	May 9.958	− 1.6	½
27	3223 B	10.721	+13.5	
28	3225 E	10.986	+ 6.6	
29	3226 A	11.722	−25.6	
30	3245 F	23.993	−23.4	
31	3340 A	July 18.745	−31.9	
32	3344 A	19.744	+ 5.6	½

Preliminary elements were computed graphically by the method of Lehmann-Filhés.[1] A first solution based on these preliminary elements gave the following values:

ELEMENTS I

$$\begin{aligned} \text{Period} &= 3.0708 \text{ days}, \\ e &= 0.0162, \\ T &= \text{J. D. } 2415368.772, \\ \omega &= 103^\circ\!.472, \\ K &= 23.39, \\ \mu^\circ &= 117^\circ\!.2334, \\ \text{Velocity of system} &= -8.45 \text{ km.} \end{aligned}$$

From these elements an ephemeris was computed and differential coefficients derived; and from these, after including as factors for homogeneity

$$\begin{aligned} x &= \delta V, \\ y &= [1.6925]\delta T, \\ z &= [4.4239]\delta\mu, \\ u &= \delta K, \\ v &= [1.3750]\delta\omega, \\ w &= [1.3718]\delta e, \end{aligned}$$

[1] *A. N.*, **136**, 17, 1894.

the following weighted equations of condition were formed:

No.	δV	δT	$\delta\mu$	δK	$\delta\omega$	δe	n
1....	$+1.000x$	$+0.022y$	$+0.020z$	$+0.996u$	$-0.038v$	$-0.187w$	$+0.253=0$
2....	+1.000	−0.440	−0.392	−0.894	+0.435	+0.648	−0.290
3....	+1.000	+0.448	+0.250	+0.885	−0.467	+0.654	−0.090
4....	+1.000	−0.887	−0.493	−0.352	+0.909	−0.128	+0.872
5....	+1.000	−0.913	−0.508	−0.259	+0.938	−0.141	+0.608
6....	+1.000	−0.194	−0.104	−0.984	+0.181	+0.164	−0.337
7....	+1.000	−0.553	−0.278	+0.808	+0.560	−0.985	−0.539
8....	+1.000	+0.779	+0.384	+0.615	−0.790	+1.000	+0.489
9....	+1.000	+0.448	+0.218	+0.885	−0.467	+0.654	−0.083
10....	+1.000	+0.772	+0.370	−0.640	−0.777	−0.909	−0.228
11....	+1.000	−0.632	−0.299	−0.755	+0.635	+0.924	−0.333
12....	+1.000	+0.838	+0.386	+0.533	−0.848	+0.981	+0.514
13....	+0.707	+0.686	−0.377	−0.182	−0.685	−0.193	−0.691
14....	+1.000	+1.000	−0.557	+0.063	−1.000	+0.362	−0.282
15....	+1.000	+0.728	−0.422	−0.689	−0.735	−0.955	+0.290
16....	+1.000	−0.287	+0.219	−0.959	+0.276	+0.356	−0.449
17....	+0.707	−0.514	+0.393	+0.449	+0.525	−0.638	−0.253
18....	+1.000	−0.278	+0.213	−0.962	+0.267	+0.337	+0.054
19....	+1.000	+0.260	−0.204	+0.960	−0.279	+0.298	+0.659
20....	+0.707	+0.477	−0.375	−0.523	−0.482	−0.698	+0.518
21....	+1.000	−0.027	+0.021	+0.996	+0.012	−0.284	−0.011
22....	+0.707	+0.704	−0.561	−0.090	−0.703	−0.007	−0.999
23....	+1.000	+0.001	−0.001	−1.004	−0.017	−0.233	+0.275
24....	+1.000	+0.121	−0.128	+0.988	−0.139	+0.014	−0.290
25....	+1.000	−0.908	+1.000	−0.279	+0.932	+0.702	+0.250
26....	+0.707	−0.629	+0.709	+0.227	+0.648	−0.290	−0.163
27....	+1.000	+0.281	−0.317	+0.954	−0.300	+0.341	−0.130
28....	+1.000	+0.738	−0.832	+0.663	−0.750	+0.992	−0.148
29....	+1.000	+0.680	−0.767	−0.736	−0.687	−0.986	+0.018
30....	+1.000	+0.700	−0.798	−0.717	−0.707	−0.975	+0.673
31....	+1.000	−0.209	+0.249	−0.980	+0.196	+0.196	−0.199
32....	+0.707	−0.533	+0.635	+0.422	+0.546	−0.608	+0.025

Whence the normal equations:

[aa]	[ab]	[ac]	[ad]	[ae]	[af]	[an]
+29.000	+ 2.623	−2.470	− 0.650	− 2.767	+ 1.119	+0.441
	+11.221	−3.064	+ 2.030	−11.374	+ 0.088	−0.441
		+6.751	+ 1.619	+ 3.097	+ 1.722	+0.020
			+17.224	− 2.086	+ 2.619	+0.194
				+11.533	− 0.146	+0.455
					+12.433	−0.325
						+5.755

The solution of these normal equations gave as corrections to Elements I:

$$\delta V = +0.09 \text{ km},$$

$$\delta T = +0.190 \text{ days},$$

$$\delta\mu = -0.0000007,$$
$$\delta K = +0.077,$$
$$\delta\omega = +0.395 \text{ radians},$$
$$\delta e = -0.0021.$$

FINAL ELEMENTS

Period $= 3.0708 \pm 0.000032$ days,

$e = 0.0141 \pm 0.0166,$

$T =$ J. D. 2415368.962 ± 0.499 days,

$\omega = 126^\circ.112 \pm 58^\circ.6,$

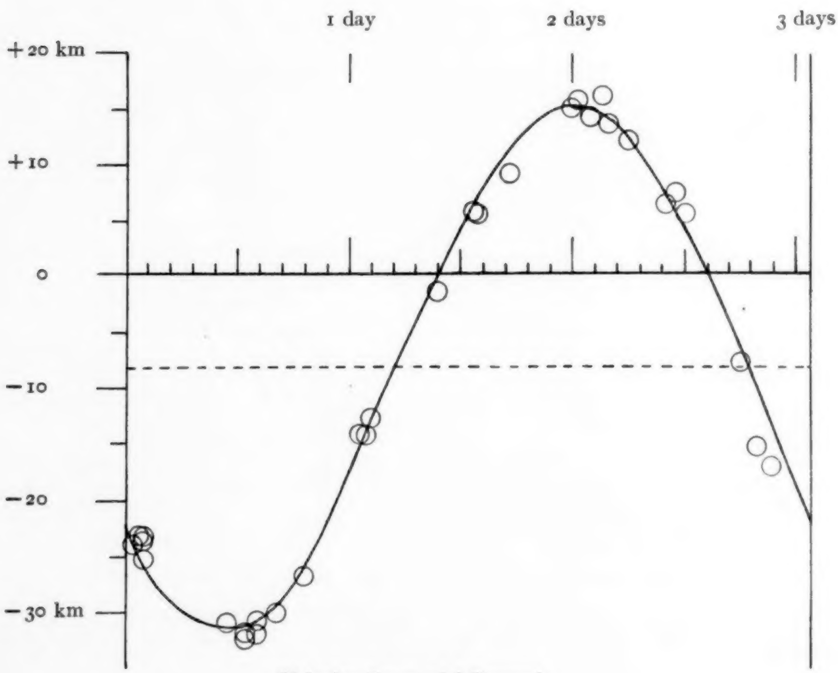
$K = 23.47 \pm 0.324,$

Velocity of system $= -8.36$ km ± 0.30 km,

$a \sin i = 9{,}900{,}000$ km.

(pvv) Eph. $= 43.36$

(pvv) Equ. $= 43.24$.



Velocity-Curve of θ *Draconis*

The probable error of a plate is 0.87 km. This is represented in the accompanying diagram of the velocity-curve and the observations by the radius of the small circles. The velocity of the center of mass of the system is given by the dotted line.

The residuals found from these elements are tabulated in the final table, together with a comparison of the change in the residuals secured respectively from the final ephemeris and by direct substitution in the equations of condition.

No.	Resid. O.−C.	Eph.−Eq.	No.	Resid. O.−C.	Eph.−Eq.
1	+0.65	−0.01	17	−1.36	+0.14
2	−0.74	−0.03	18	+0.27	−0.10
3	−0.20	−0.09	19	+1.82	−0.04
4	+2.16	+0.08	20	+2.01	−0.05
5	+1.40	+0.13	21	−0.13	+0.03
6	−0.83	−0.09	22	−3.94	−0.04
7	−1.78	+0.15	23	+0.86	−0.10
8	+1.40	−0.10	24	−0.83	−0.03
9	−0.19	−0.08	25	+0.46	+0.08
10	−0.65	−0.07	26	−1.05	+0.06
11	−0.92	−0.06	27	−0.36	−0.04
12	+1.45	−0.02	28	−0.37	−0.09
13	−2.74	−0.04	29	+0.04	−0.05
14	−0.82	−0.02	30	+1.85	−0.05
15	+0.70	+0.04	31	−0.43	+0.11
16	−1.12	−0.04	32	−0.29	+0.02

THE D. O. MILLS EXPEDITION
Santiago, Chile, June 1907

ORBIT OF THE SPECTROSCOPIC BINARY *a CARINAE*

By HEBER D. CURTIS

The binary character of this star ($\alpha = 9^h\, 8^m.4$; $\delta = -58^\circ\, 33'$) was discovered by Professor W. H. Wright in the course of the work of the D. O. Mills Expedition to the Southern Hemisphere.[1] It is of visual magnitude 3.5, and the exposure time used has been fifty to sixty minutes under average observing conditions. It contains, in the part of the spectrum covered by the spectroscope of the Mills Reflector, only the following six lines:

λ 4267.316 *C*
4340.634 *H*
4388.100 *He*
4437.718 *He,* generally rather faint
4471.646 *He*
4481.400 *Mg*

The star is given as Type *B3A* in the Harvard classification, and the lines are of quite fair quality for this type of spectrum.

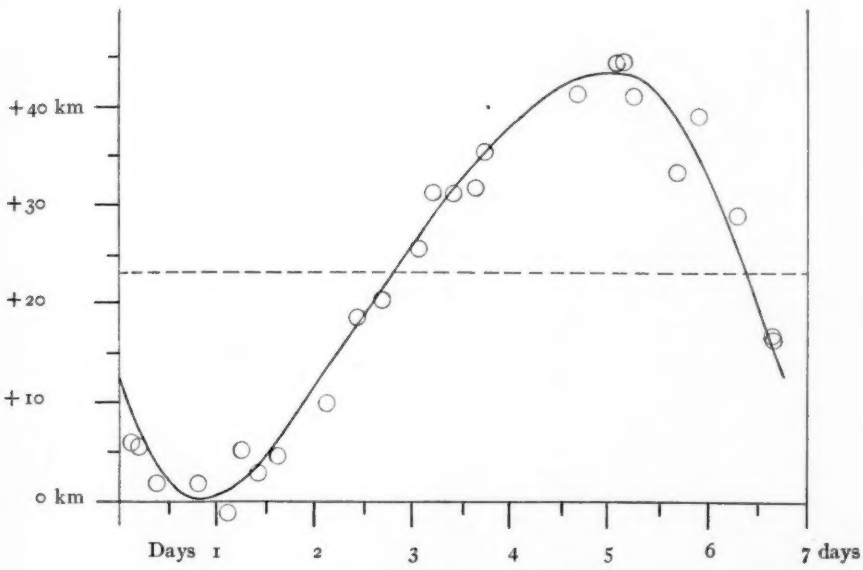
The orbit depends upon the following twenty-five plates:

No.	Plate	G. M. T.	Velocity	Measurer	O.–C.
			km		km
1.......	199 I	1904, Feb. 29.671	+5.5 +6.0	Wright Palmer	−3.4
2.......	570 II	1905, Jan. 30.683	+32.6 +33.8	Wright Palmer	−5.1
3.......	588 II	Feb. 9.640	+9.2 +10.9	Wright Palmer	−3.2
4.......	607 III	22.617	+4.2 +4.8	Wright Palmer	−1.4
5.......	617 II	Mar. 7.577	−1.2	Palmer	−2.0
6.......	911 IV	1906, Mar. 30.578	+3	Curtis	+0.9
7.......	1071 III	1907, Jan. 15.789	+39.0	Curtis	+4.8
8.......	1077 II	19.814	+31.1	Curtis	+2.4
9.......	1084 II	21.755	+44.4	Curtis	+1.6
10.......	1102 II	25.810	+18.6	Curtis	+0.7
11.......	1107 II	26.786	+31.2	Curtis	−0.4

[1] *Lick Observatory Bulletin*, **3**, 111, 1905; *Astrophysical Journal*, **21**, 374, 1905.

No.	Plate	G. M. T.	Velocity	Measurer	O.-C.
			km		km
12.......	1128 IV	1907, Feb. 2.751	+31.8	Curtis	−2.4
13.......	1140 III	5.768	+16.2	Curtis	+1.6
14.......	1145 III	6.665	+ 2.0	Curtis	+1.8
15.......	1151 III	19.734	+ 1.9	Curtis	−2.1
16.......	1162 IV	Mar. 2.739	+41.3	Curtis	−1.3
17.......	1167 IV	4.736	+16.7	Curtis	+1.9
18.......	1183 III	14.625	+25.7	Curtis	−1.0
19.......	1188 III	16.634	+44.3	Curtis	+1.3
20.......	1195 III	19.574	+ 5.2	Curtis	+3.3
21.......	1199 II	23.528	+40.8	Curtis	−2.3
22.......	1208 IV	24.582	+28.8	Curtis	+3.4
23.......	1282 II	April 30.489	+20.4	Curtis	−0.9
24.......	1294 II	May 1.496	+35.2	Curtis	+0.1
25.......	1319 II	11.476	+ 5.7	Curtis	−1.2

An orbit was computed graphically from these values by the method of Lehmann-Filhés.[1] These elements were then tested and changed by varying the elements after comparison with the observed velocities



Velocity-Curve of *a Carinae*

[1] *A. N.*, **136**, 17, 1894.

so as to give as close as possible a representation of the observations. The following elements resulted:

$$\begin{aligned}
\text{Period} &= 6.744 \text{ days,} \\
T &= \text{J. D. } 2416533.81, \\
\omega &= 115^\circ\!.84, \\
K &= 21.5, \\
\mu^\circ &= 53^\circ\!.380, \\
e &= 0.18. \\
\text{Velocity of system} &= +23.3 \text{ km,} \\
a \sin i &= 1{,}960{,}000 \text{ km.}
\end{aligned}$$

A least-square solution would not be warranted by the number and character of the lines available for measurement.

In the accompanying figure I have plotted the separate observations with the orbit curve, the dotted line representing the velocity of the center of mass of the system. The actual residuals, in the sense observed minus computed, are given in the last column of the table. While some of these are rather large, they are not excessive when the character and number of the lines used is taken into account.

THE D. O. MILLS EXPEDITION
Santiago, Chile, June 1907

ORBIT OF THE SPECTROSCOPIC BINARY κ *VELORUM*

By HEBER D. CURTIS

The binary nature of κ *Velorum* ($\alpha = 9^h\ 19.0^m$; $\delta = -54°\ 35'$) was discovered by Professor W. H. Wright in the work of the D. O. Mills Expedition to the Southern Hemisphere.[1] The star is given in the Harvard classification as Type $B3A$; its visual magnitude is 2.6. The following six lines are the only ones usable in the portion of spectrum given by the spectrograph of the Mills Expedition:

λ 4267.316 *C*
4340.634 *H*
4388.100 *He*
4437.718 *He*
4471.646 *He*
4481.400 *Mg*

Of these lines the helium line at λ 4437 is generally faint and was not usable on a number of the plates. In addition to these lines exceedingly faint traces of a number of the oxygen lines of the β *Crucis* type are discernible on a few of the plates; these lines were never distinct enough to use.

The orbit depends upon the following twenty-seven plates:

No.	Plate	G. M. T.	Velocity	Measurer	O − C.
			km		km
1.......	216 I	1904, Mar. 6.739	+70.0 / +66.8	Palmer / Wright	+2.4
2.......	535 II	1905, Jan. 14.703	+12.9	Palmer	−2.0
3.......	602 II	Feb. 20.651	+65.7	Palmer	−0.7
4.......	618 III	Mar. 7.601	+53.3	Palmer	−3.6
5.......	1052 III	1907, Jan. 11.844	+58.6	Curtis	+2.3
6.......	1057 II	12.788	+57.9	Curtis	−1.1
7.......	1065 II	14.829	+58.5	Curtis	−2.7
8.......	1072 IV	15.824	+64.8	Curtis	+2.6
9.......	1085 III	21.788	+65.8	Curtis	−0.8
10.......	1129 II	Feb. 2.790	+62.0	Curtis	+0.4
11.......	1194 II	Mar. 19.534	−21.0	Curtis	−0.5
12.......	1198 II	20.556	−19.2	Curtis	+0.2

[1] *Lick Observatory Bulletin*, **3**, 111, 1905; *Astrophysical Journal*, **21**, 374, 1905.

No.	Plate	G. M. T.	Velocity	Measurer	O.-C.
			km		km
13.......	1200 III	1907, Mar. 23.570	−15.2	Curtis	+0.4
14.......	1207 III	24.545	−14.5	Curtis	+0.5
15.......	1256 III	April 20.591	+33.8	Curtis	+3.3
16.......	1264 III	25.572	+38.2	Curtis	−0.8
17.......	1270 III	26.555	+43.2	Curtis	+2.6
18	1283 III	30.480	+46.7	Curtis	−0.2
19.......	1300 II	May 5.494	+52.7	Curtis	−1.4
20.......	1364 III	June 14.466	+22.1	Curtis	+3.7
21.......	1366 II	19.463	+ 0.3	Curtis	+0.3
22.......	1377 I	22.470	− 7.6	Curtis	+1.0
23.......	1384 I	23.479	− 8.8	Curtis	+2.5
24.......	1390 I	24.463	−13.3	Curtis	+0.5
25.......	1397 I	26.457	−19.2	Curtis	−1.4
26.......	1402 I	July 1.451	{−28.2 −29.9}	Curtis Curtis	−4.8
27.......	1408 I	2.460	{−23.9 −25.1}	Curtis Curtis	+0.2

A preliminary orbit was computed graphically from these values in accordance with the method of Lehmann-Filhés.[1] Then a number of sets of elements were tested by comparison with the observations, slight changes being made in the values given by the graphical solution. The resulting orbit which best satisfies the observations is as follows:

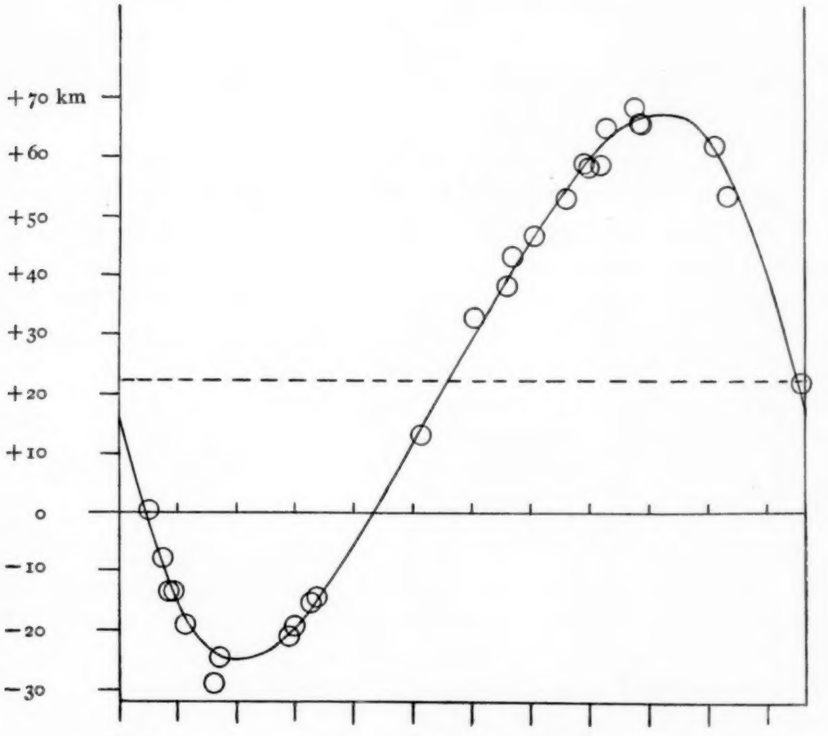
ELEMENTS OF κ *VELORUM*

$$
\begin{aligned}
\text{Period} &= 116.65 \text{ days}, \\
e &= 0.19, \\
K &= 46.5, \\
T &= \text{J. D. } 2416459.00, \\
\omega &= 96\overset{\circ}{.}23. \\
\text{Velocity of system} &= +21.9 \text{ km}, \\
a \sin i &= 73{,}200{,}000 \text{ km}.
\end{aligned}
$$

These elements are represented by the curve of the accompanying diagram, where the dotted line gives the velocity of the center of mass of the system. The total range in the velocity is ninety-three kilometers. It is to be expected that future observations may change the

[1] *A. N.*, **136**, 17, 1894.

value of the period slightly, as the observation period covers only about fourteen revolutions of the system. An ephemeris computed



Velocity-Curve of κ *Velorum*

from these elements gives the residuals, observed minus computed, which are tabulated in the last column of the table of observations.

The D. O. Mills Expedition
Santiago, Chile, August 1907

ORBIT OF THE SPECTROSCOPIC BINARY α *PAVONIS*

By HEBER D. CURTIS

The binary character of α *Pavonis* ($\alpha = 20^{h}\ 17^{m}.7$; $\delta = -57^{\circ}\ 0'.3$) had been suspected by Professor W. H. Wright in Chile from preliminary measures of the first four plates taken, and has been independently discovered from the definitive reductions of the same plates made by Dr. S. Albrecht at Mt. Hamilton. The star is of the type *B3A*, similar to *α Carinae* and *κ Velorum*, though the lines are doubtless somewhat better than in these stars. Its visual magnitude is 2.0. Under fair observing conditions satisfactory plates can be secured in twenty-two to twenty-six minutes.

The following twenty-two plates form the basis of the elements derived in this paper:

No.	Plate	G. M. T.	Velocity	Measurer	O.−C.
			km		km
1.......	17 I	1903, Sept. 23.582	+ 2.0	Palmer	−0.9
2.......	333 II	1904, May 29.909	+ 9.5	Albrecht	+0.3
3.......	756 IV	1905, Aug. 3.697	− 0.9	Albrecht	+1.9
4.......	785 III	25.676	− 4.2	Albrecht	+1.1
5.......	935 II	1906, Oct. 7.571	+ 3.3 + 2.1	Paddock Curtis	−1.1
6.......	966 II	Nov. 6.514	+ 0.4 + 1.5	Paddock Curtis	−1.4
7.......	1292 III	1907, April 30.924	− 1.5 + 0.9	Paddock Curtis	+0.7
8.......	1323 III	May 11.831	− 3.8	Curtis	−0.2
9.......	1336 IV	13.880	+ 4.5	Curtis	+0.1
10.......	1343 IV	14.886	+ 7.3	Curtis	−0.3
11.......	1349 II	18.835	+ 4.5	Curtis	+1.3
12.......	1382 II	June 22.884	+ 2.9	Curtis	−1.1
13.......	1400 IV	26.764	− 5.8	Curtis	−0.7
14.......	1407 II	July 1.886	+10.4	Curtis	+1.3
15.......	1411 IV	2.685	+ 8.7	Curtis	−0.3
16.......	1418 II	3.768	+ 7.3	Curtis	+0.5
17.......	1428 IV	6.804	− 2.4	Curtis	+1.2
18.......	·1435 III	19.742	− 5.8	Curtis	−0.5
19.......	1441 IV	20.767	− 4.0	Curtis	+0.4
20.......	1445 II	25.645	+ 7.7	Curtis	−1.5
21.......	1451 I	27.778	+ 5.9	Curtis	+0.7
22.......	1456 II	29.630	− 2.6	Curtis	−1.1

The lines upon which the above radial velocities depend are the six characteristic lines of this type, no others being visible in this region of the spectrum.

λ 4267.316 *C*
4340.634 *H*
4388.100 *He*
4437.718 *He*
4471.646 *He*
4481.400 *Mg*

A set of preliminary elements was first derived graphically by the method of Lehmann-Filhés.[1] Changes were then made in the derived elements, after comparing with the curve given by the observations, and several sets of elements tested by the observation values. It is the opinion of the writer that the application of the method of least squares to stars of this type of spectrum and number of lines will not be warranted, except in the case that a large number of observations are available extending over a long interval of time. With some experience in the method it is possible in a relatively short time to test and change the elements given by the graphical solution until the resulting values would be little if any bettered by a least-square solution. The computation of even three or four test ephemerides involves much less labor and time than a least-square solution.

By such methods the following set of elements was decided upon as best satisfying the data furnished by the observed radial velocities:

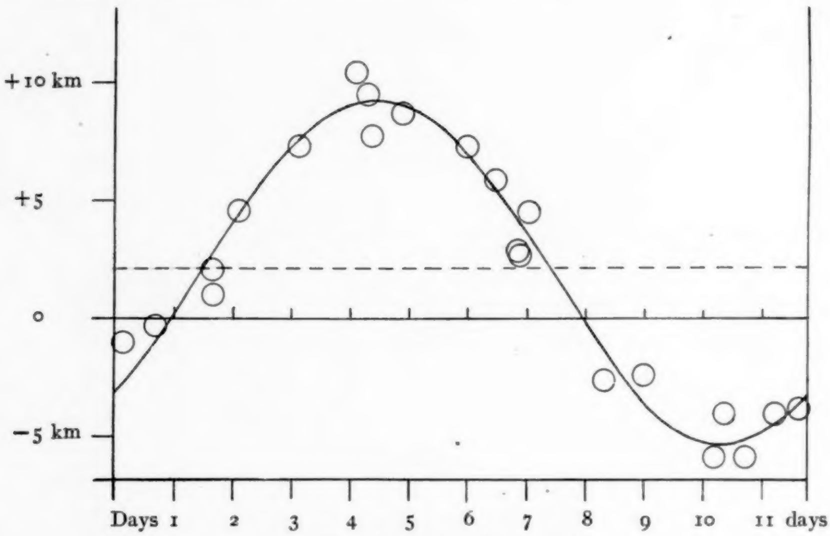
ELEMENTS OF α PAVONIS

Period = 11.753 days,
$e = 0.01$,
$K = 7.25$,
$T =$ J. D. 2416379.90,
$\omega = 224\overset{\circ}{.}80$.
Velocity of system = +2.0 km,
$a \sin i = 1{,}170{,}000$ km.

The total range in the radial velocity is only 14.5 kilometers. The observations are about as well satisfied by a circular orbit, as the eccentricity is evidently very small.

[1] *A. N.*, **136**, 17, 1894.

This velocity-curve and the separate observations are plotted in the accompanying diagram, the dotted line, as usual, representing the velocity of the center of mass of the system. The numerical values



Velocity-Curve of *α Pavonis*

of the residuals secured by comparison of the observed radial velocities with an ephemeris computed from these elements is given in the final column of the observation table.

THE D. O. MILLS EXPEDITION
Santiago, Chile, August 1907

DEFINITIVE ORBIT OF THE SPECTROSCOPIC BINARY *ω DRACONIS*

By ARTHUR B. TURNER

The spectroscopic binary *ω Draconis* was discovered by Director Campbell and announced by him in the *Astrophysical Journal* in August 1899 (**10**,179). It is an *F*-type star with rather broad, fuzzy lines.

The orbit depends on the following twenty-six plates taken with the Mills Spectrograph, which have been measured and reduced by the writer:

No.	Date G. M. T.	Observed Velocity	O.−C. Preliminary Orbit	O.−C. Final Orbit	Comparison of Residuals
		km	km	km	km
1	1899, July 25.776	+19.2	+1.54	+0.62	+0.01
2	Aug. 8.779	−45.8	+0.25	+0.24	−0.06
3	9.774	−11.7	+0.21	−0.42	+0.03
4	29.721	−48.5	−0.08	+0.06	−0.04
5	1906, June 29.852	+11.8	+1.50	+1.16	−0.06
6	July 3.735	+10.6	−0.05	−0.22	−0.04
7	10.867	− 8.9	−2.03	−1.99	+0.05
8	11.851	−42.8	+0.30	+0.51	+0.06
9	15.731	+10.6	+1.58	+1.27	−0.04
10	16.722	−31.2	−0.82	−0.65	+0.03
11	22.720	−49.9	−1.55	−1.30	−0.01
12	23.798	−32.4	−0.92	−0.39	−0.01
13	25.799	+19.8	−0.84	−1.44	−0.05
14	26.807	−11.7	−0.55	−0.57	±0.00
15	29.767	− 3.8	−0.90	−0.66	+0.10
16	30.753	+21.6	−0.60	−1.20	−0.01
17	Aug. 1.786	−38.5	−0.40	−0.23	±0.00
18	6.765	−28.0	−0.47	−0.35	+0.01
19	7.759	−47.6	+1.74	+2.04	−0.02
20	9.759	+14.4	+1.71	+1.50	+0.07
21	13.768	−37.6	−0.97	−0.42	+0.01
22	16.713	− 3.5	−1.17	−1.35	−0.09
23	Sept. 6.712	+ 3.7	+1.29	+1.07	+0.05
24	1907, July 28.773	−14.2	+0.95	+1.51	+0.04
25	Aug. 5.771	−15.1	+0.63	+0.54	−0.01
26	7.751	−38.6	−0.92	−0.29	−0.02
	[*pvv*]		26.782	22.678	

By the method of Lehmann-Filhés[1] preliminary elements were obtained, but owing to the fact that the orbit was so nearly a circle,

[1] *Astronomische Nachrichten*, **136**, 17, 1894.

some difficulty was encountered in getting satisfactory values for ω and T. The above preliminary residuals were obtained by diminishing ω by 270° and making a corresponding change in T and giving a small value to the eccentricity. The following are the

PRELIMINARY ELEMENTS

$$
\begin{aligned}
\text{Velocity of system} &= -13.65 \text{ km},\\
T &= 1906 \text{ July } 23.285,\\
&= \text{Julian Day } 2417385.285,\\
e &= 0.0058,\\
\omega &= 319^\circ\!.837,\\
\log \mu &= 0.07558,\\
\mu &= 68^\circ\!.1866,\\
K &= 35.80,\\
\text{Period} &= 5^d\!.27963,\\
a \sin i &= 2{,}599{,}000 \text{ km}.
\end{aligned}
$$

The differential coefficients were then computed from these elements and the equations of condition formed. The epoch, Julian Day 2416329.359, was used in computing the coefficients of $\delta\mu$. A sixth unknown with the coefficient unity was introduced to allow for change in the velocity of the system. The equations were then weighted and the coefficients made homogeneous by the use of the following factors:

$$
\begin{aligned}
x &= \delta V && = \delta V\\
y &= \frac{K\mu}{(1-e^2)^{\frac{3}{2}}}\delta T && = 42.61\ \delta T\\
z &= 1466.4\frac{K}{(1-e^2)^{\frac{3}{2}}}\delta\mu && = 52500\ \delta\mu\\
u &= \delta K && = \delta K\\
v &= K\delta\omega && = 35.80\ \delta\omega\\
w &= K\delta e && = 35.80\ \delta e\\
\log \text{unit error} &= 0.2405 && = 0.2405
\end{aligned}
$$

The following equations were then obtained:

							Wt.
$+1.000x$	$-0.498y$	$-0.498z$	$+0.875u$	$+0.496v$	$+0.948w$	$-0.885=0$	1
$+1.000$	-0.414	-0.411	-0.905	$+0.420$	$+0.013$	$-0.144=0$	1
$+1.000$	-1.008	-0.998	$+0.049$	$+1.003$	-0.705	$-0.121=0$	1
$+1.000$	-0.217	-0.212	-0.972	$+0.223$	$+0.416$	$+0.046=0$	1
$+1.000$	$+0.747$	-0.541	$+0.669$	-0.743	-0.730	$-0.862=0$	1
$+1.000$	-0.748	$+0.544$	$+0.678$	$+0.743$	$+0.571$	$+0.029=0$	1
$+0.707$	$+0.691$	-0.506	$+0.134$	-0.692	-0.667	$+0.825=0$	½
$+1.000$	$+0.562$	-0.408	-0.822	-0.559	$+0.880$	$-0.172=0$	1
$+1.000$	$+0.777$	-0.571	$+0.633$	-0.773	-0.790	$-0.908=0$	1
$+1.000$	$+0.872$	-0.642	-0.469	-0.877	$+0.114$	$+0.471=0$	1
$+1.000$	$+0.230$	-0.170	-0.969	-0.228	$+0.973$	$+0.891=0$	1
$+1.000$	-0.867	$+0.642$	-0.498	$+0.869$	-0.941	$+0.529=0$	1
$+1.000$	$+0.302$	-0.224	$+0.958$	-0.296	$+0.256$	$+0.483=0$	1
$+1.000$	$+0.992$	-0.737	$+0.071$	-0.994	-0.840	$+0.316=0$	1
$+1.000$	-0.966	$+0.719$	$+0.298$	$+0.960$	-0.271	$+0.517=0$	1
$+1.000$	-0.087	$+0.065$	$+1.000$	$+0.090$	$+0.864$	$+0.345=0$	1
$+1.000$	$+0.718$	-0.536	-0.683	-0.723	$+0.602$	$+0.230=0$	1
$+1.000$	$+0.911$	-0.683	-0.389	-0.916	-0.060	$+0.270=0$	1
$+1.000$	-0.003	$+0.003$	-0.996	$+0.007$	$+0.760$	$-1.000=0$	1
$+1.000$	-0.690	$+0.521$	$+0.735$	$+0.686$	$+0.696$	$-0.983=0$	1
$+1.000$	-0.762	$+0.576$	-0.643	$+0.766$	-0.760	$+0.557=0$	1
$+0.707$	$+0.667$	-0.507	$+0.223$	-0.669	-0.703	$+0.475=0$	½
$+1.000$	$+0.894$	-0.690	$+0.448$	-0.892	-0.975	$-0.741=0$	1
$+0.707$	-0.712	$+0.707$	-0.030	$+0.709$	-0.583	$-0.386=0$	½
$+1.000$	$+0.990$	-0.989	-0.059	-0.994	-0.674	$-0.362=0$	1
$+1.000$	-0.737	$+0.737$	-0.672	$+0.741$	-0.708	$+0.529=0$	1

These gave the following normal equations:

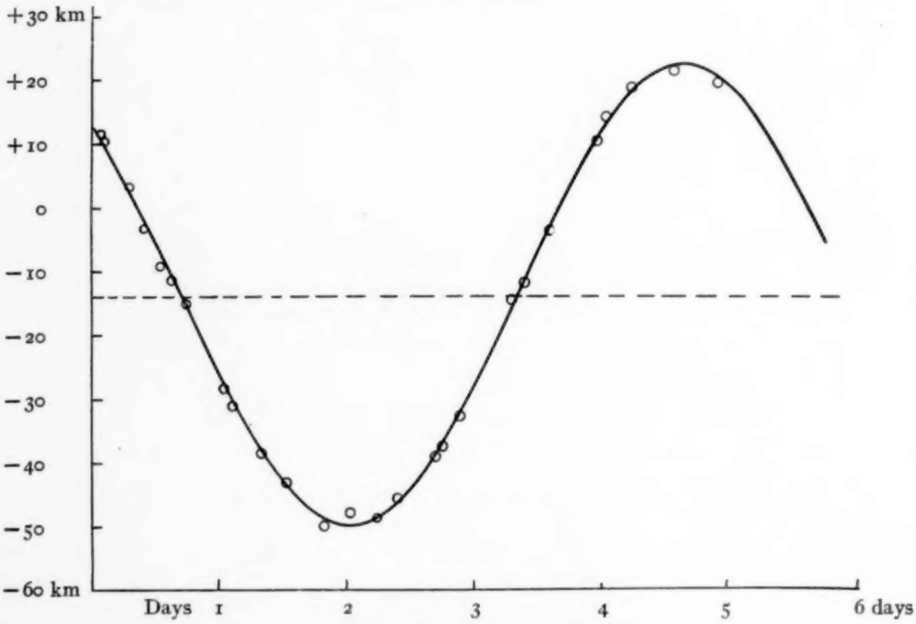
$24.499x$	$+1.455y$	$-4.719z$	$-1.432u$	$-1.452v$	$-1.742w$	$-0.319=0$
	$+13.255$	-7.804	$+0.149$	-13.254	-1.437	$-0.350=0$
		$+8.874$	-0.074	$+7.810$	$+1.261$	$+1.447=0$
			$+11.221$	-0.163	-0.799	$-2.833=0$
				$+13.255$	$+1.438$	$+0.345=0$
					$+12.453$	$-0.898=0$

The solution supplied the following corrections to the elements:

$$\delta V=-0.0316 \text{ km},$$
$$\delta T=+0.198 \text{ days},$$
$$\delta\mu=-0.0000107 \text{ radians},$$
$$=-0^\circ.0006,$$
$$\delta K=+0.459,$$
$$\delta\omega=+0.243 \text{ radians},$$
$$=+13^\circ.924,$$
$$\delta e=+0.00491.$$

The probable error of a single observation is ±0.75 kilometer per second. The final elements, with their probable errors, are as follows:

$$\begin{aligned}
\text{Velocity of System} &= -13.68 \pm 0.16 \text{ km},\\
T &= 1906 \text{ July } 23.493 \pm 0.394 \text{ days},\\
&\quad \text{Julian Day } 2417385.493,\\
e &= 0.0107 \pm 0.0060,\\
\omega &= 333^{\circ}.761 \pm 26^{\circ}.9,\\
\log \mu &= 0.0755725 \pm 0.0000027,\\
\mu &= 68^{\circ}.1860 \pm 0^{\circ}.0004,\\
K &= 36.26 \pm 0.24,\\
\text{Period} &= 5.27968 \pm 0.00003 \text{ days},\\
a \sin i &= 2,632,300 \text{ km}.
\end{aligned}$$



Velocity-Curve of *ω Draconis*

The comparison of the residuals obtained from the final elements with those obtained from substitution in the equations of condition is shown in column six of the table. The small differences indicate that a second solution is unnecessary.

The final velocity-curve is represented by the accompanying diagram, the observed places as given by the plates being represented by small circles. The velocity of the center of mass of the system is represented by the dotted line.

In conclusion I wish to thank Director Campbell for putting at my disposal the necessary facilities of the Observatory for carrying on the above investigation.

MOUNT HAMILTON
August 31, 1907

THE SPECTROSCOPIC BINARY η *VIRGINIS*

By NAOZO ICHINOHE

The variability of the radial velocity of this star was discovered at this observatory and also independently at the Lick Observatory. The investigation of its orbit was undertaken by me two years ago at the suggestion of Mr. Frost, to whom my sincere thanks are due; and the material has now become tolerably sufficient for the discussion of the principal star. The number of spectroscopic binaries is small in which the fainter component is strong enough to give a well measurable spectrum; and the velocity-curve for the faint component has been published in hardly any instance. As Messrs. Frost and Adams have pointed out, η *Virginis* shows a pretty intense spectrum of the faint component on the three-prism spectrograms. As the number of lines which are well measurable is comparatively large, the resulting velocity for the component is pretty accurate. This is why I feel so interested to investigate this star, but the number of plates taken with the full dispersion of the Bruce spectrograph is not sufficient for the satisfactory discussion of the faint component; some plates were taken with the single prism, but on such plates I was not able to see the lines of the spectrum of the faint component.

The whole number of the spectrograms so far obtained at this observatory is 25, among which 16 plates were taken with the three-prism spectrograph and the remaining 9 plates with the single prism. The following journal of observations of η *Virginis* is similar to those for which I have already determined the spectroscopic orbits. It is to be noticed that the column of temperature is slightly different from others in that in the cases of the single-prism spectrograms, the temperature inside the outer case of the spectrograph is given, but in the cases of the three-prism plates, the reading of the thermometer inserted in the inner case is recorded.

Among the plates, on IB 1041 the comparison spectrum was too weak and the two last named are very weak and not suitable for accurate measurement.

Plate	Date	G. M. T.	Exposure	Slit-Widths mm	Temp.	Observer	Seeing
B 487....	1903, Jan. 14	23^h 17^m	95^m	0.046	− 6°3 C.	A	3; 2
A 388....	1903, Jan. 16	22 14	132	0.046	− 0.3	A	3; 3
B 493....	1903, Feb. 4	23 12	120	0.046	− 4.9	F	3; 2
A 399....	1903, Feb. 5	20 47	108	0.046	− 6.1	A	3; 3
B 539....	1903, Dec. 13	22 46	120	0.051	−18.3	F	4; 2
B 551....	1904, Feb. 19	21 11	108	0.036	− 7.6	F	4; 4
IB 488....	1905, Jan. 21	21 12	45	0.038	− 9.9	B	3; 2
B 580....	1905, Feb. 27	19 47	120	0.044	+ 0.2	FB	3; 2
B 626....	1906, Jan. 5	21 33	124	0.046	− 2.4	B	3; 3
B 650....	1906, Mar. 16	21 59	120	0.051	− 7.8	F	3; 3
B 651....	1906, Mar. 19	18 54	180	0.051	− 3.9	F	2; 2
B 657....	1906, April 16	19 28	120	0.059	+11.0	B	4; 3
IB 742....	1906, April 23	16 35	46	0.051	+ 9.7	F	3; 3
B 664....	1906, May 18	16 45	120	0.046	+22.8	B	4; 2
IB 926....	1906, Dec. 14	22 12	53	0.051	+ 0.8	B	3; 2
B 685....	1906, Dec. 23	23 07	130	0.057	− 7.3	Fox	4; 3
B 700....	1906, Dec. 28	21 08	135	0.057	+ 1.7	B	2; 2
IB 950....	1907, Jan. 21	22 53	46	0.051	−10.4	F	2; 2
IB 962....	1907, Jan. 25	23 10	45	0.051	−14.4	B	3; 3
IB 989....	1907, Feb. 18	21 12	44	0.051	+ 2.4	B	3; 3
IB1001....	1907, Feb. 22	22 00	45	0.051	− 9.0	B	3; 3
IB1013....	1907, April 1	14 52	48	0.046	+ 1.4	B	2; 3
IB1041....	1907, April 26	17 26	63	0.051	+ 3.6	F	3; 3
B 717....	1907, June 14	15 28	120	0.040	+22.5	F	3; 3
B 720....	1907, June 16	15 12	120	0.051	+26.7	B	2; 3

The examination of the plates shows that the star belongs to the later stage of the *Orion* type, or more properly to the first stage of the *Sirian* type. The lines are very narrow and well defined, so that they are very well measurable. The general character of the spectrum of the star may be stated as follows. The hydrogen lines become exceedingly narrow and intense, and many metallic lines are very well developed, especially those of iron and titanium. The helium lines do not appear, or they are very weak. The magnesium line λ 4481 is as strong as $H\gamma$. The silicon lines λλ 4128 and 4131 are well seen, but I could not see other lines belonging to the same element. The spectrum of the faint component is an exact duplicate of that of the bright component. This might show that both components must have a very close relation to each other; probably they have a common origin and constitute a system.

The measurement of the plate A 388 was made by Mr. Adams; plate B 493 was measured by both Messrs. Frost and Adams, and the measurement of A 399 was again made by Mr. Adams alone. These

results can be seen in their announcement of the discovery.[1] The results of two plates by the Lick observers can be found in the same journal.[2] I have measured all the remaining plates obtained here of the star. The following table contains the results of the measurements. As usual, the first column gives the plate numbers; the second the epochs of the observations in Julian days, two decimal places being retained. The third gives the measured velocities reduced to the sun. The fourth column shows the number of lines which I have measured for the determination of the velocity on each plate. The results for the plates B 717 and B 720 will be considered only as the approximate values, and I have omitted them from the discussion except in the case of the faint component.

η Virginis

Plate No.	Julian Day	v	n	Phase	v_c	$v-v_c$
B 487......	2416129.97	−27.6	9	0.0	−27.8	+0.2
A 388......	6131.92	−31.5	14	1.9	−31.4	−0.1
B 493......	6150.97	+ 0.4	16	21.0	+ 1.1	− 0.7
A 399......	6151.87	+ 3.4	16	21.9	+ 3.0	+0.4
B 539......	6462.95	+19.0	14	45.4	+18.9	+0.1
B 551......	6530.88	+19.9	13	41.4	+19.6	+0.3
IB 488......	6867.88	− 0.5	7	18.9	− 2.2	+1.7
B 580......	6904.83	+ 5.4	12	55.9	+10.4	−5.9
B 626......	7216.90	−30.2	14	8.4	−30.8	+0.6
B 650......	7286.92	−33.7	15	6.6	−33.4	−0.3
B 651......	7289.79	−27.0	16	9.4	−28.9	+1.9
B 657......	7317.81	+18.9	12	37.4	+19.1	−0.2
IB 742......	7324.69	+22.4	13	44.3	+19.3	+3.1
B 664......	7349.70	−24.2	14	69.3	−21.1	−3.1
IB 926......	7559.92	− 0.6	13	63.9	− 5.4	+4.8
B 685......	7568.96	−29.5	16	1.0	−29.6	+0.1
B 700......	7573.88	−32.1	14	5.9	−33.8	+1.7
IB 950......	7597.91	+20.0	15	30.0	+14.7	+5.3
IB 962......	7601.97	+20.3	9	34.0	+17.7	+2.6
IB 989......	7625.88	+10.3	18	57.9	+ 7.6	+2.7
IB 1001......	7629.92	+ 1.3	17	62.0	− 0.8	+2.1
IB 1013......	7667.62	+12.6	6	27.8	+12.4	+0.2
IB 1041......	7692.72	+ 7.2	9	52.9	+14.2	−7.0
B 717......	7741.65	+21.7	7	29.9	+14.8	.....
B 720......	7743.63	+26.4	7	32.9	+17.1	.....

The lines and their normal wave-lengths which are used for the star are as follows. In the table, the last column shows how many times the corresponding line has been used for the star, the whole number of the plates being 25.

[1] *Astrophysical Journal*, **17**, 150, 1903. [2] *Ibid.*, **18**, 307, 1903.

Element	λ	n	Element	λ	n
Ca, K	3933.825	4	—	4385.548	3
Fe	3936.965	1	*Ti*	4387.007	1
Fe	4005.408	2	*Ti*	4395.201	12
Ti	4012.541	2	*Ti–Cr*	4399.935	3
Ti	4024.726	1	*Fe*	4404.927	5
Ti	4028.497	2	*Cr*	4411.240	1
Fe–Ti	4030.646	1	—	4416.985	2
Fe	4045.975	8	*Ti*	4417.884	1
Fe–Ti	4053.981	1	*Ti+Fe*	4427.420	1
Fe	4063.759	5	*Ti*	4443.976	18
Fe	4065.537	1	*Ti*	4468.663	19
Fe+Fe	4067.248	1	*Fe*	4472.884	1
Fe	4071.908	3	*Mg*	4481.400	25
Sr	4077.885	3	*Ti*	4488.493	1
Ti	4078.631	1	*Ti*	4489.262	1
Hδ	4101.890	1	—	4491.570	2
Si	4128.211	1	*Ti*	4501.445	19
Si	4131.047	1	*Fe?*	4508.455	12
Fe	4132.235	1	—	4515.508	5
Ti; Cr–	4163.818	3	*Fe?*	4520.397	10
—	4171.854	1	—	4522.802	11
Fe	4173.480	1	*Fe*	4528.798	1
Fe	4202.198	2	*Ti?*	4529.656	1
Fe	4215.581	2	*Ti*	4534.139	12
Mn–Fe	4233.328	1	*Cr*	4541.690	3
Fe	4271.934	1	*Fe+Ti*	4549.767	22
Fe	4294.301	1	*Ba*	4554.211	8
Ti	4302.085	1	—	4556.063	4
Fe	4308.081	2	*Cr?*	4558.827	5
Ti	4313.034	1	*Ti*	4563.939	10
Ti	4315.138	1	*Co–Fe*	4565.842	1
Ti	4325.939	3	*Ti*	4572.156	10
Hγ	4340.634	8	*Fe*	4584.018	4
Cr	4351.930	4	—	4588.381	1
Ti	4367.839	1	*Ti*	4590.126	1
Fe	4383.720	6	*Ti*	4629.521	1

The investigation of the period of the oscillation was made by myself in the spring of 1906. At that time the material was small, but after numerous trials, I found a value of $71^{d}.9$ for the period. For this determination, the two Lick plates were of great service. The value satisfied all the results known at that time very well. The results obtained later do not change this value, so $71^{d}9$ was taken as the period for the present discussion.

Let us first consider the principal component. At the beginning the plates were taken with the full dispersion and the preliminary velocity-curve was drawn with all the data known. The curve and the observations were accordant. The two Lick plates were also accordant. But when the plates began to be taken with the single-

prism spectrograph, it became necessary to change the nature of the curve if we assume that there is no systematic difference between both series of observations made with the three-prism and single-prism spectrograph. And if we do so, the result is that the three-prism results lose their good accordance, especially the plate B 580, and the residuals become larger than when we take the curve depending merely upon those results. I already suspected such systematic error in the measures of μ *Sagittarii*, but unfortunately the question is not settled. If there exists some difference of this nature, we cannot use both series of observations without the danger of introducing some errors in the computed elements of the orbit, unless we know the nature of the systematic difference and apply the proper corrections to the observations. If we had sufficient material to discuss the star independently for both series, it would be interesting, but we have not the data needed. Thus I was obliged to assume that we can use both kinds of observations equally well, notwithstanding that I suspect some differences.

Now, the fifth column of the above table was calculated with the period $71^{\mathrm{d}}.9$ and assuming $0^{\mathrm{d}}0$ phase for the epoch J. D. $2416129^{\mathrm{d}}.97$. Then, as usual, the column v was taken for the ordinates and the column Phase for the abscissas. All the observations being platted in this way, a smooth curve was drawn through or near to these points. The curve enabled me to determine the radial velocity of the center of gravity of η *Virginis* as follows:

Radial velocity of the center of gravity $= -0.4$ km.

Thus we can say that the center of gravity of this stellar system is at rest referred to the sun.

The examination of the curve gave then the following values for calculating the elements according to the method of Lehmann-Filhés:

$$A = 20.0 \text{ km}, \qquad B = 33.6 \text{ km},$$
$$z_1 = 738, \qquad z_2 = -738,$$
$$t_1 = 61^{\mathrm{d}}.9, \qquad t_2 = 92^{\mathrm{d}}.0.$$

We now compute the following elements:

$$U = 71^{\mathrm{d}}.9,$$
$$u_1 = 75^{\circ}\ 18',$$

$$\omega = 180^\circ\ 0',$$
$$e = 0.254,$$
$$\mu = 5^\circ\!.01,$$
$$\text{or } \log \mu = 8.9414,$$
$$T = 76^d\!.96,$$
$$\text{or } T = J.D.\ 2416206.93,$$
$$a \sin i = 25{,}290{,}000 \text{ km},$$
$$m + m' = \frac{0.13\ \odot}{\sin^3 i}\ .$$

If the inclination be not quite small, *a* as well as $m+m'$ will assume the following values in astronomical units.

i	a	$m+m'$
30°	0.3402	1.02
45°	0.2406	0.36
60°	0.1950	0.19
75°	0.1761	0.14
90°	0.1701	0.13

To see how the set of elements will represent the observations I have computed an ephemeris using these elements. The sixth column of the above table shows these computed values, and in the last column, the differences between the observed and computed values, $v - v_c$, are given. We see that the elements represent the observations pretty well, but for the plates B 580 and B 664 we have the residuals -5.0 and -3.1 km respectively. These are comparatively great for the results obtained with the three-prism spectrograph. Mr. Campbell did not give the times of his observations, so that we cannot obtain exact phases for them, but when we assume that he observed the star on the meridian, then the residuals will be $+1.2$ and -3.2 km respectively. In Fig. 1 the curve is drawn with the computed values, and the single circles show the values as observed with the one-prism arrangement of the spectrograph; the double circles show the values observed with three prisms; the Lick results are represented by the circles with black centers.

Let us next examine the faint component of *η Virginis*. The results of the observations of the component are shown in the follow-

ing table. In the column under the head of phase we assumed the same value of the period as for the other component.

η Virginis (Second Component)

Plate	v	n	Phase	v_c	$v-v_c$
B 487	+39 km	8	0.0	+58 km	−19 km
A 388	+42	4	1.9	+59	−17
B 493	+62	6	21.0	+39	+23
B 539	− 8	7	45.4	− 2	− 6
B 551	−29	12	41.4	− 5	−24
B 580	+67	12	55.9	+26	+41
B 626	+41	12	8.4	+58	−18
B 650	+41	15	6.6	+60	−19
B 651	+43	16	9.4	+58	−15
B 657	−36	12	37.4	− 4	−32
B 664	+36	12	69.3	+55	−19
B 685	+47	10	1.0	+58	−11
B 700	+45	10	5.9	+60	−15
B 717	−41	6	29.9	+ 6	−35
B 720	−34	7	32.9	+ 1	−33

These are platted in Fig. 2, which shows that the velocity of the faint component varies with the same period $71^d.9$; but the nature

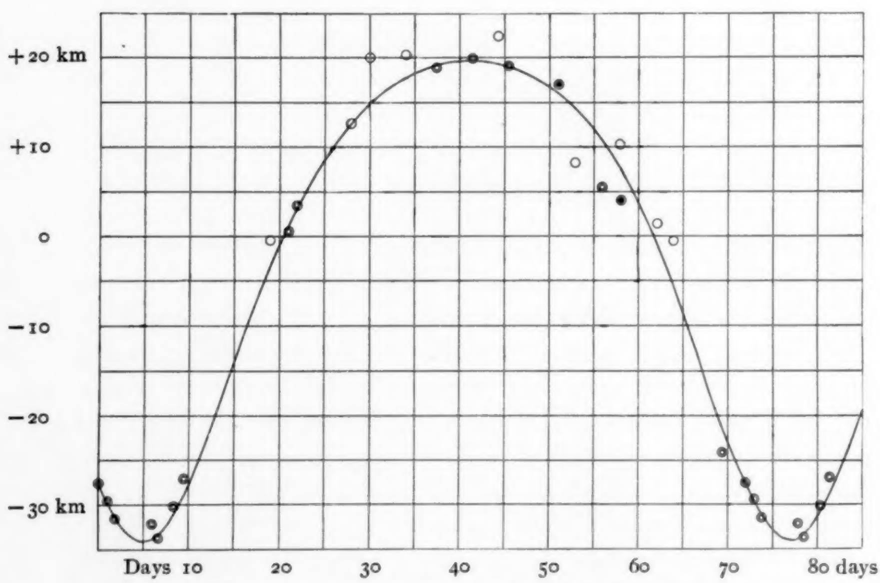


FIG. 1.—Velocity-Curve of η Virginis, Brighter Component

of the curve is not simple as the other component. Although we have not sufficient observations for this component, still it would probably be safe to draw the velocity-curve as shown in the figure. The curve permits the two following conclusions at once: (1) the radial velocity of the center of gravity of the faint component does not coincide with that of the principal one; and (2) the curve is not simply periodic with the value 71^d9 but it is affected by another cause whose

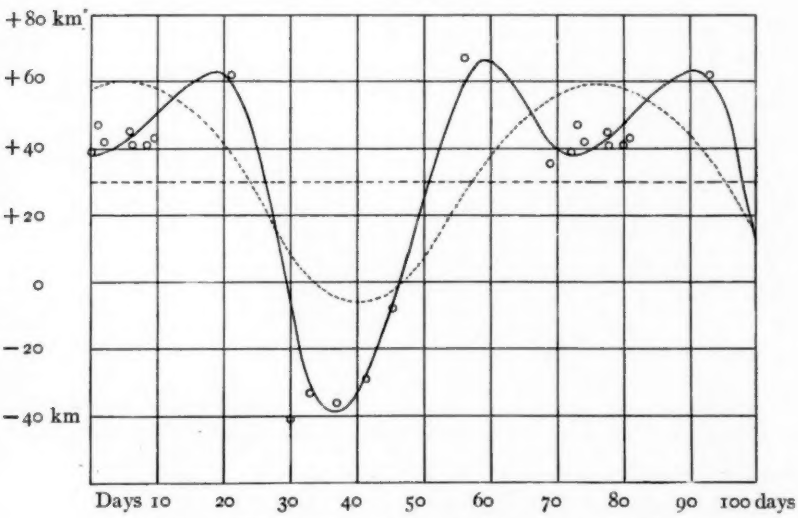


FIG. 2.—Velocity-Curve for Second Component of η Virginis

action is also periodic commensurably with the main period. From these results, I think the faint component is not only a companion to the principal star but that there is at least another component nearer to the principal than the faint component.

To discuss these points more fully, I first determined the radial velocity of the center of gravity of this component and the result is +30 km. This indicates that the faint component is receding from the bright component with a velocity of about 2,166,000 km in a day in the line of sight. If this continues for quite a long time, these two components cannot remain a mechanical system. But our observations cover only a little more than four years so that we cannot conclude anything as to this. Still, we may suppose that the faint

component is situated very far from the bright one and revolves around the central sun with a long period so that the radial velocity obtained for the center of gravity is simply the projection of the orbital velocity in the line of sight. Then, the oscillation of the radial velocity of the component may be looked upon as a perturbation by the principal component or its nearer companion. If such supposition be correct after a longer series of observations of this faint

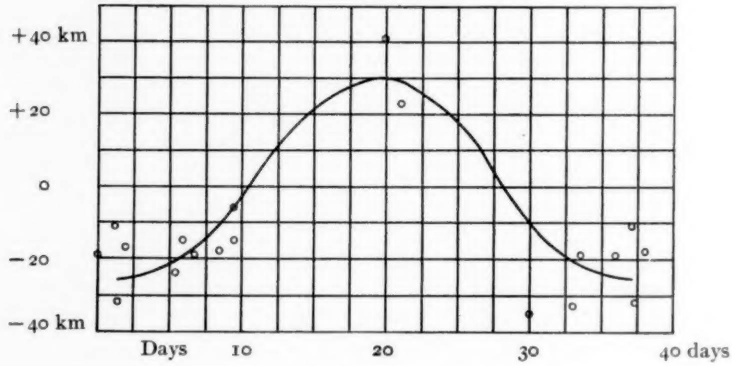


FIG. 3

component, covering many decades, it would be possible to know more precisely about the system of η *Virginis*.

To get a better conception of the perturbation, I have drawn a mean curve, Fig. 3, balancing the smaller disturbances in such way that the areas inclosed, above and below, by both the curve and the observed curve become equal, and they cancel out when we assign a + sign for the area above and a − sign for that below. The curve is dotted in the figure. Now finding the residuals, the observed values minus the computed, we have the fifth column of the last table. Finally, examining these we see that the residuals have a period of $35^{d}.95$, or exactly one-half of the main period $71^{d}.9$. The third figure shows the curve drawn with the residuals as the ordinates and the phases as abscissas. Such irregularity was found in the cases of ζ *Geminorum*, *W Sagittarii* and also other short-period variables. We do not know yet what is the real cause of this oscillation. Mr. Roberts gave an explanation of such disturbance and found that when we are concerned with bodies whose dimensions are comparable with that of the

distance between the centers of the components, we must expect such periodic oscillation in the cases where the bodies are not spherical. The above are simply inferences which we can reach at present, but of course the true nature of the mystery remains without a proper explanation.

YERKES OBSERVATORY
August 1907

EIGHT STARS WHOSE RADIAL VELOCITIES VARY

By W. W. CAMPBELL and J. H. MOORE

The following six stars have been shown to have variable radial velocities. The plates of dates earlier than June 1903 were taken with the Mills spectrograph for which minimum deviation was set at $H\gamma$, while plates of later dates were obtained with the remounted Mills spectrograph whose minimum deviation is set at λ 4500. The observations are not distributed in such a way as to give a definite idea of the period of any of these binary systems.

1 o Tauri ($\alpha = 3^h\ 19^m.4;\ \delta = +8°\ 40'$)

Plate	Date	Velocity	Measured by
965 C................	1898, September 23	−17 km	Campbell
		−18.7	Burns
974 B................	1898, September 28	−16	Campbell
		−16.9	Burns
1449 D................	1899, September 6	−18	Campbell
		−21.7	Burns
3841 F................	1904, October 4	−25	Moore
		−23.7	Newkirk
4052 A................	1905, October 9	−22	Moore
		−20.8	Newkirk
4428 E................	1906, September 16	−20	Moore
		−19.2	Newkirk
4837 D................	1907, August 2	−15	Moore

The spectrum of this star is of the *K* type. Its variable radial velocity was suspected by Mr. Moore in 1904 and confirmed by the measures of recent plates. The period is probably long.

5 f Tauri ($\alpha = 3^h\ 25^m.4;\ \delta = +12°\ 35'$)

Plate	Date	Velocity	Measured by
517 A................	1897, October 19	+15.2 km	Wright
		+14	Campbell
531 C................	1897, October 28	+14.9	Burns
		+12	Reese
998 A................	1898, October 10	+10.7	Burns
		+11	Campbell
1010 D................	1898, October 17	+12.2	Burns
		+11	Campbell
1022 C................	1898, October 19	+11.6	Burns
3051 E................	1903, November 29	+ 9	Curtis
3530 D................	1904, November 7	+22	Moore
4843 D................	1907, August 5	+27	Moore

The type of spectrum is *K*. The period of this binary is probably long. Its variable velocity was discovered by Mr. Moore.

7 Camelopardalis ($\alpha = 4^h\ 49^m.3$; $\delta = +53°\ 35'$)

Plate	Date	Velocity	Measured by
2584 A................	1902, November 4	−40 km	Burns
		−33	Curtis
		−20	Curtis
3072 E................	1903, December 6	−17	Moore
4612 B................	1907, February 7	+20.5	Moore
4643 D................	1907, February 27	− 3.2	Moore
4647 B................	1907, March 13	− 1.8	Moore
4683 A................	1907, April 22	+22.5	Campbell
4861 D................	1907, August 8	+23	Moore

The spectrum is of type *A*. The magnesium line λ 4481 is good on all of the plates and on some of them the iron line λ 4549 is also measurable. The binary character of this star was shown by Mr. Moore from a measure of the third plate, and is confirmed by measures of later plates. Its period is undoubtedly short.

A *Bootis* ($\alpha = 14^h\ 13^m.8$; $\delta = +35°\ 58'$)

Plate	Date	Velocity	Measured by
4234 B................	1906, May 31	−40 km	Moore
		−38.9	Newkirk
4631 D................	1907, February 10	−12	Moore
4656 A................	1907, March 29	− 8	Moore
4680 C................	1907, April 21	−11	Campbell
4730 B................	1907, May 29	−18.6	Campbell

The type of spectrum is *I*. The first, fourth, and fifth plates are underexposed. However, many of the lines are easily measurable on these plates, so that there is no doubt but that the variation is real. The period is probably short. The variable velocity was discovered by Mr. Moore from the second plate.

β Coronae ($\alpha = 15^h\ 23^m.7$; $\delta = +29°\ 27'$).

Plate	Date	Velocity	Measured by
1717 D................	1900, April 18	−15 km	Wright
		−17.7	Burns
1728 B................	1900, May 13	−16	Wright
		−18.0	Burns
2121 F................	1901, May 6	−20	Reese
		−18.0	Burns
2339 D................	1902, February 13	−17	Reese
		−15.0	Burns
3775 D................	1905, April 11	−23.5	Moore
4655 E................	1907, March 29	−33	Moore
4661 D................	1907, April 8	−30	Moore
4804 A................	1907, July 17	−28	Moore

The spectrum of this star is a very good *F* type.

The binary character was discovered by Mr. Moore. The period is probably long.

62 ξ Cygni ($\alpha = 21^h\ 1^m.3$; $\delta = +43°\ 32'$)

Plate	Date	Velocity	Measured by
3347 D................	1904, July 19	−19.6 km	Newkirk
3522 A................	1904, October 31	−19	Campbell
		−18.1	Newkirk
4830 B................	1907, July 29	−24	Campbell
		−24.1	Newkirk
4870 A................	1907, August 12	−22	Moore
		−22.7	Newkirk
4933 A................	1907, September 15	−14	Moore

The spectrum is of type *K*, with very good lines. The variation is small, but the spectrum admits of accurate measurement, and there can be no doubt that the variation is a real one. The variable velocity of this star was discovered by Mr. Campbell. Its period is probably long.

Since the above list of spectroscopic binaries went to press, the following two stars have been shown to have variable velocities in the line of sight.

d Tauri ($\alpha = 4^h\ 30^m.2$; $\delta = +9°\ 57'$)

Plate	Date	Velocity	Measured by
4121 A................	1905, November 18	+ 52.2 km	Moore
4475 C................	1906, October 1	+102.0	Moore
4896 F................	1907, August 25	+ 66.5	Moore
4972 E................	1907, October 6	− 33.5	Moore
4995 E................	1907, October 13	− 45.2	Moore

The spectrum is a fair *F* type with rather broad but easily measurable lines. The variation in velocity was discovered by Mr. Moore. Its period is probably short.

ζ Cephei ($\alpha = 22^h\ 7^m.4$; $\delta = 57°\ 43'$)

Plate	Date	Velocity	Measured by
830 C................	1898, July 20	−18 km	Campbell
842 B................	1898, July 25	−17.4	Burns
1053 B................	1898, November 1	−18.5	Burns
1089 B................	1898, November 14	−18 −18.6	Campbell Burns
2209 D................	1901, July 31	−21 −21.2	Reese Burns
4402 B................	1906, September 2	−20 −19	Campbell Moore
4803 D................	1907, July 16	−14	Moore
4986 E................	1907, October 10	−16	Moore

The type of spectrum is *K*. The variable velocity was shown by Mr. Moore from the measures of recent plates.

LICK OBSERVATORY
August 20, 1907
October 23, 1907

TWO STARS WHOSE RADIAL VELOCITIES ARE VARIABLE

By W. H. WRIGHT

Spectrograms secured by the D. O. Mills Expedition to the Southern Hemisphere show the radial velocities of the following stars to be variable.

x Carinae ($\alpha = 11^h\ 4^m.4$, $\delta = -58°\ 26'$)

Date	Velocity	Measured by
1904, January 5..........	+17.1 km	R. H. Curtiss
1904, April 16............	+14.5	Wright
1905, January 6..........	+ 8.9	Wright
1905, February 24........	+ 7.4	Wright
1905, June 21............	+15.6	Wright
1906, February 24........	+17.4	Wright
1907, March 5...........	+ 4.5	Paddock
1907, April 27...........	+ 3.3	Paddock
1907, May 13............	+ 3.7	Paddock

The lines in the spectrum of this star are somewhat diffuse and difficult of measurement. The variation was strongly suspected from the first six observations, and is amply confirmed by Mr. Paddock's measures of plates kindly secured by Professor H. D. Curtis.

ι Gruis ($\alpha = 23^h\ 4.7^m$, $\delta = -45°\ 47'$)

Date	Velocity	Measured by
1903, November 9........	−10.0 km	Wright
	− 9.7	Albrecht
1904, September 12.......	− 5	Palmer
	− 2.3	Wright
1904, October 27.........	− 4	Palmer
	− 5.6	Wright
1905, November 1........	− 3.8	Albrecht
	− 3.6	Wright
1905, November 13.......	− 3.8	Palmer
1905, November 19.......	− 3.0	Albrecht
1907, June 23............	−18.8	H. D. Curtis

The last observation, by Dr. Curtis, confirms the variation suspected from the preceding measures.

MT. HAMILTON
September 19, 1907

NOTE ON THE CAUSE OF THE PRESSURE-SHIFT OF SPECTRUM LINES

By W. J. HUMPHREYS

It was long ago suggested by Fitzgerald[1] that the increase in the specific inductive capacity of a gas, due to an increase in its density, is a *vera causa* for at least a part of the pressure-shift of spectrum lines; and very recently Larmor[2] made the same claim and showed that "if the vibrator operates as a simple Hertzian doublet," then, under certain reasonable assumptions, "the dielectric influence of the neighboring molecules is a *vera causa* of the right order of magnitude."

This theory is very pretty and I trust it will be worked up more completely, because if true it must provide for all the pressure effects, while a failure to do so will tend to prove that the vibrator is not of the nature of the simple Hertzian oscillator.

It appears safe to assume that the period of any vibrating body is dependent upon the elasticity both of the body itself and of the surrounding medium that takes up its vibrations, and therefore a change in either of these elasticities will change the period. In all such cases, if the inertia remains constant, we have the equation $et^2=k$, a constant, where e is the elasticity, and t the period. Therefore in the case of the vibrator that produces a spectrum line, any decrease in e causes a corresponding increase in λ^2. Besides, the greater λ, the less its increase necessary to produce a given increase in its square.

Consequently if the source of a spectrum line is a kind of Hertzian doublet, and its pressure-shift due to increase in the specific inductive capacity of the surrounding medium, it appears that in general we should expect among other results due to pressure:

a) A shifting of the entire line to the red.

What we get by experiment is a broadening of the line, both to the violet and to the red, with the latter predominating.

b) The increase of λ^2 to be a linear function of the pressure.

Unfortunately the change in λ is too small to test this relation.

[1] *Astrophysical Journal*, **5**, 210, March 1897.

[2] *Ibid*, **26**, 120, September 1907.

Let $\lambda_1 - \lambda_0$, or $\Delta\lambda$, be the change in wave-length produced by a change in pressure from p_0 to p_1, then

$\lambda_1^2 - \lambda_0^2 = 2\lambda_0\Delta\lambda + (\Delta\lambda)^2$, or simply $2\lambda_0\Delta\lambda$ to within the limits of experimental error, since $\Delta\lambda$ is always very small.

But $\Delta\lambda$ is approximately a linear function of the increase in pressure, and therefore so also is $2\lambda_0\Delta\lambda$, or $\lambda_1^2 - \lambda_0^2$ since $(\Delta\lambda)^2$ is negligible in comparison with the other term.

c) The greater the inductive capacity of the gas used, the greater the shift for any given pressure.

This conclusion is not yet established; it demands a knowledge, difficult to obtain, of the inductive capacity of the interior of the arc itself.

d) That the greater λ the less its shift.

Experiment does not give any well-marked relation between wave-length and pressure-shift, but the trend undoubtedly is in the other direction; that is, for the shifts to be greater in the case of lines of longer wave-length.

Possibly these and all other objections can be met by properly distributing, between the interior of the atom itself and its surrounding medium, the elasticity that determines the period of any given line. But this makes the problem a very complex one, and it seems doubtful whether it can ever be made to fit the facts of experiment as well as do magnetically interacting Saturnian atoms.

I fully agree with Larmor that the shift of spectrum lines probably is not strictly a pressure-effect, though it increases directly with the pressure of the surrounding gas. But I cannot at present agree with him in calling it a density-effect, since this would ascribe to heavy atoms an influence directly proportional to their mass, a result by no means experimentally established—in fact the masses of the neighboring atoms seem to be of secondary importance. Possibly the term proximity-effect might better suit the facts of experiment, as this refers to compactness of numbers without regard to their individual masses, and therefore while proportional to pressure is different from density.

Mount Weather Observatory
Bluemont, Va.
October 1907